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Three-dimensional simulation of a Laddermill

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Abstract. This paper describes a program for simulation of Laddermill, a novel concept for sustainable energy production. Laddermill is a flexible multi-body structure consisting of the kites and the cable; thus its mathematical model includes models of both. Kites are assumed rigid so that equations of motion are written only for their centers of masses, the model of elastic string is used for the cable. Fully three-dimensional equations of motion are used. The result of this project is a program for simulating Laddermill which can be used for further research.

Keywords. Laddermill, multi-body dynamics, sustainable.

List of notations

a is acceleration

A is the cross-section area of a cable element

c_D, c_L are aerodynamic coefficients of drag and lift

D is drag

E is the elastic modulus of a cable element

g is gravity acceleration vector with coordinates $g = (0, 0, g)$

i is the index of a coordinate (x is 1, y is 2, z is 3), it is the second index of the two

j is the index of a kite (cable element), it is the first index of the two

l is lift unit vector

L is lift

m is mass

N is the number of kites (cable elements)

r is radius-vector with coordinates $r_j = (r_{j1}, r_{j2}, r_{j3})$

R is the difference of radius vectors of two adjacent cable elements or a kite and cable element

S is projected area

t is time

T is tension

V is velocity vector relative to the airflow with coordinates $V_j = (v_{j1}, v_{j2}, v_{j3})$

W is wind velocity vector with coordinates $W = (w_1, w_2, w_3)$

α is attack angle

ϕ, θ, ψ are angles of roll, pitch and yaw

ρ is the density of air

Introduction

The concept for sustainable energy production called Laddermill is known for 10 years now [1] and refers to the system of kites on one rope that drives the generator as kites pull it. The benefits of this approach to energy production is a low weight and low cost of the structure and simplicity of maintenance [2, 3]. Preliminary theoretical investigation promises capabilities of a vast power output [4]. However this kite system requires effective flight control of the kites to produce energy and such control is impossible without understanding of dynamics of the Laddermill as a system. The literature on dynamics of cable systems is extremely wide, starting from untraceable publications of classics of mathematical physics. An extensive literature review on this subject can be found in [5]. However, there are only a few publications now about the dynamics Laddermill: internal reports [6, 7] about O-Mill Laddermill simulation program written by Professor W.J. Ockels and a his paper [8] are the only available sources of information for now. Thus, the purpose of this study is creating a tool for investigation of Laddermill dynamics that can be further incorporated into a Laddermill control software.

Methods

The Laddermill is a flexible multi-body structure consisting of the kites and the cable. Thus, mathematical model of Laddermill consists of mathematical models of kites and of the cable. In the simulation of movement of the kites presented in this paper only their centres of gravity are considered. The cable is considered elastic. Fully three dimensional equations of motion are used. The result of this work is the Laddermill simulation program that can be used for improving energy production and ensuring the safety of Laddermill operation.

As mentioned above, Laddermill consists of a ground station, a cable and kites. Thus, models of both kites and the cable should be present in the resulting mathematical model. They are simulated in the boundaries of the same flight dynamics approach, but with certain differences. The mathematical model of the kite will be explained first as more complicated. The mathematical model of the cable will be explained by referring to the model of the kite.

Mathematical model of the kite

Two reference frames are used for this simulation. The basic one is a ground-fixed reference frame "E" (see fig.1), all coordinates and velocities are calculated in it. It is the same for the whole Laddermill. Zero is in the point where ground station is situated, vertical axis points down. Body-fixed reference frame "B" (see fig.2) is used for tracking the attitude of each kite (or cable element) over time.
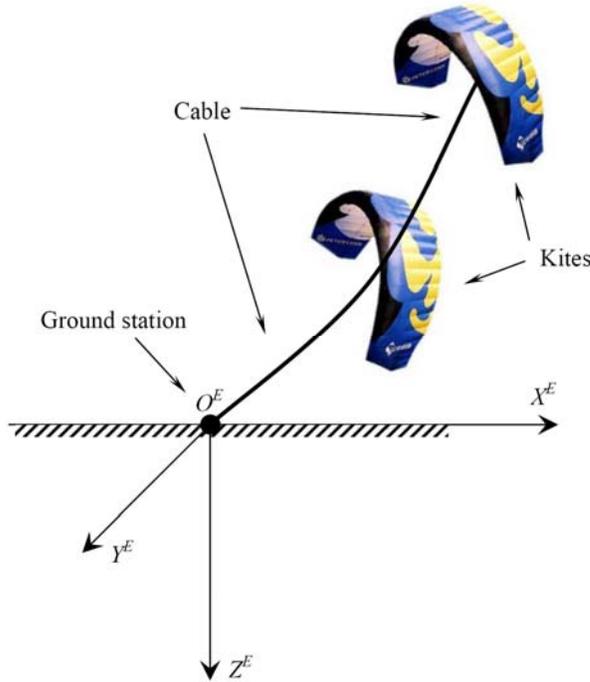

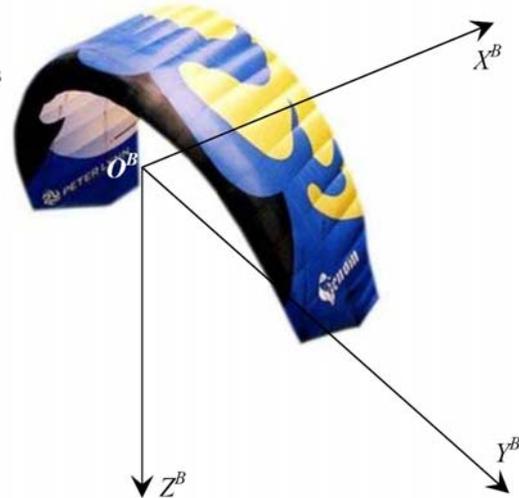Fig. 1. Earth-fixed reference frame ("E")   Fig. 2. Body-fixed reference frame ("B")

Let's number the kites so that the one that is closest to the ground will have number 1 and the highest one will have the number N. The index for kite's number is j.

The position of the kite is represented by altitude $-z$ and two other coordinates x, y. For the simplicity of processing the coordinates have been combined into a position vector $\boldsymbol{r}$ pointing from the ground station to the kite's centre of mass:

$$\boldsymbol{r}_j = \left(x_j, y_j, z_j\right) = \left(r_{j1}, r_{j2}, r_{j3}\right) \tag{1}$$

The attitude of the kite is represented by angles of pitch θ, roll ϕ and yaw ψ. These are also the control angles. For now it is assumed that all kites are fully controllable and can have exact attitude that is set by remote control on the ground station.
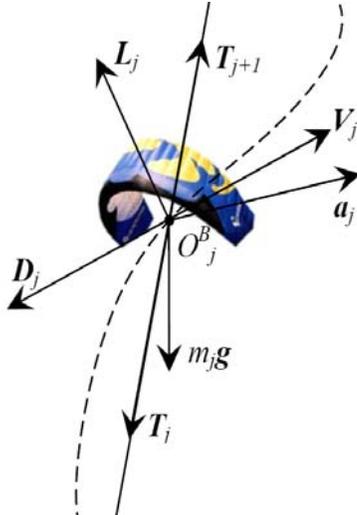
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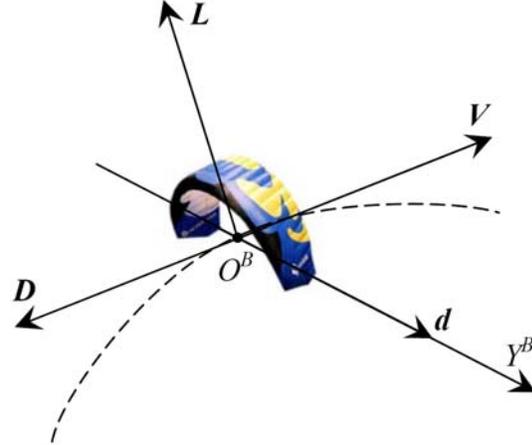
Fig. 3. Forces acting on kite

Fig. 4. Direction of lift and drag

Forces acting on kite

The flying kite is affected by gravity, tension of the cable and aerodynamic forces and moments of which only lift and drag will be considered in the current study as most common (see Fig. 3):

$$m\boldsymbol{a}_j = m_j\boldsymbol{g} + \boldsymbol{D}_j + \boldsymbol{L}_j + \boldsymbol{T}_j + \boldsymbol{T}_{j+1} \qquad (2)$$

Let's write projections of each force separately before this system of equations will be projected on coordinate axes.

The lift of the kite is perpendicular to kite's velocity $\boldsymbol{V}$ and to its wingspan direction $\boldsymbol{d}$ (see Fig. 4):

$$\boldsymbol{l} = \frac{\boldsymbol{V} \times \boldsymbol{d}^E}{V}, \qquad (3)$$

Here $\boldsymbol{V}$ is velocity, $\boldsymbol{l}$ is a unit vector of lift and $\boldsymbol{d}^E$ is a unit vector in the wingspan direction:

$$V_j = \sqrt{\sum_{i=1}^{3} v_{ji}}, \qquad (4)$$

$$\boldsymbol{l} = \frac{\boldsymbol{L}}{L}, \qquad (5)$$

$$\mathbf{d}^E = (0,1,0)^B. \qquad (6)$$

The procedure described in short by (6) is actually a transformation of vector's coordinates from reference frame B and into E [9]:

$$d_1^E = \cos\phi\cos\psi d_1^B + \cos\phi\sin\psi d_2^B - \sin\phi d_3^B, \qquad (7)$$

$$d_2^E = (\sin\theta\sin\phi\cos\psi - \cos\theta\sin\psi)d_1^B + (\sin\theta\sin\phi\sin\psi + \cos\theta\cos\psi)d_2^B + \sin\theta\cos\phi d_3^B, \qquad (8)$$

$$d_3^E = (\cos\theta\sin\phi\cos\psi + \sin\theta\sin\psi)d_1^B + (\cos\theta\sin\phi\sin\psi - \sin\theta\cos\psi)d_2^B + \cos\theta\cos\phi d_3^B. \qquad (9)$$

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The drag force is straight opposite to velocity:

$$\boldsymbol{D} = -\boldsymbol{v}D \, . \tag{10}$$

The tension is directed straight between the kite and nearest cable element:

$$\boldsymbol{R}_j = \boldsymbol{r}_j - \boldsymbol{r}_{j-1} \, , \tag{11}$$

$$R_j = \sqrt{\sum_{i=1}^{3} \left(r_{ji} - r_{j-1i} \right)^2} \, , \tag{12}$$

$$\boldsymbol{T}_j = \frac{\boldsymbol{R}_j}{R_j(t_0)} E_j A_j \, , \tag{13}$$

here zero time in brackets refers to initial conditions.

Kite's equations of motion

After dividing (2) by mass and projecting it on coordinate axes the equations of motion of the kite in Earth-fixed reference frame are defined:

$$L_{ji} = \frac{1}{2} \rho(r_{j3}) S_j c_{Lj}(\alpha_j) V_j (d_{ji+1} v_{ji+2} - d_{ji+2} v_{ji+1}) \, , \tag{14}$$

$$D_{ji} = -\frac{1}{2} \rho(r_{j3}) S_j c_{Dj}(\alpha_j) V_j v_{ji} \, , \tag{15}$$

$$T_{ji} = \frac{E_j A_j \left(R_j - R_j(t_0) \right) r_{ji}}{R_j(t_0) \sqrt{\sum_{i=1}^{3} r_{ji}^2}} \, , \tag{16}$$

$$\frac{dv_{ji}}{dt} = \frac{1}{m_j} \left(D_{ji} + L_{ji} - T_{ji} + T_{j+1i} + g_i(r_{j3}) \right), \tag{17}$$

$$\frac{dr_{ji}}{dt} = v_{ji} + w_i(r_{j3}) \, , \tag{18}$$

$$i = 1 .. 3, \quad j = 1 .. N. \tag{19}$$

Initial conditions for solution of these equations are the position $\boldsymbol{r}_j(t_0)$ and velocity $\boldsymbol{V}_j(t_0)$ of kites and ground station. The system of equations (14) – (19) is then solved using one of known methods, e.g., method of Euler.

Mathematical model of the cable

The mathematical model of elastic string is used for the cable. It describes the dynamics of elastic continuous medium with negligible bending and torsion stiffness (see fig. 5) [10]:

$$\frac{\partial \boldsymbol{V}}{\partial t} = \frac{\oiint_{\Gamma} \boldsymbol{\sigma} \cdot d\boldsymbol{n} + \iiint_{G} \boldsymbol{F} dG}{\iiint_{G} \rho_{cable} dG} \, , \tag{20}$$

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here ρ_{cable} is the density of the cable, G is the volume of the cable, Γ is its surface and $\boldsymbol{n}$ is the normal to its surface; σ are the stresses on the surface of the cable and F are the forces in its volume.
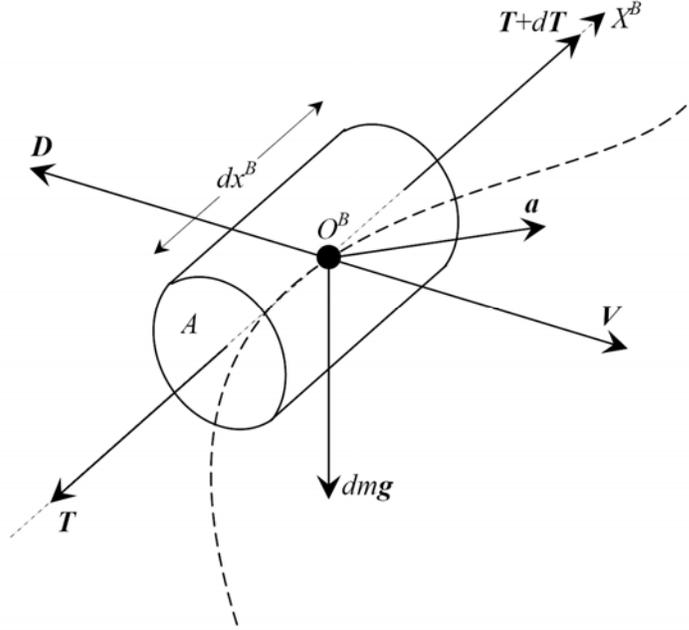

Fig. 5. Forces acting on the element of the cable

Substitution of the factual loads on the cable transforms the equation (20) into (21)

$$dm\boldsymbol{a} = \boldsymbol{D} + \boldsymbol{T} + (\boldsymbol{T} + d\boldsymbol{T}) + dm\boldsymbol{g} \qquad (21)$$

which is then solved using finite difference method. The cable is divided into N elements, each having the length h. The coordinates of the elements are $\boldsymbol{r}_j$ and velocities are $\boldsymbol{V}_j$:

$$\hat{v}_{ji} = v_{ji} + \frac{\Delta t}{m_j}\left(D(V_j, \alpha_j) - T(r_{j-1}, r_j) + T(r_j, r_{j+1}) + g_i(r_{j3})\right). \qquad (22)$$

here $^\wedge$ denotes the next moment of time. It is clear that this equation is similar to the equation (16) without lift if the latter is solved using Euler's method. Thus, the simulation of the cable as a continuous medium is actually programmed in the boundaries of the same approach (13)–(18) with the following adjustments:

- Lift force is not considered,
- The distance between close cable elements R is the step over coordinate: $R = h$.
- The movement of the cable up and down is simulated by adding or removing elements of the cable in its beginning.

Results and discussion

After performing the necessary tests the model have been used for simulation of Laddermill's flight. The identical approach to simulation of kites and pieces of cable between them allows saving a lot of computer memory: for example, a simulation of 5 km long cable with 1 cm step with 100 kites requires only 82 MB.

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Conclusion

The 3D program for simulation of Laddermill has been created within the framework of two assumptions:

- only aerodynamic coefficients of lift and drag are used for kites;
- the model of elastic string is used for the cable.

It allows simulation of Laddermill's behaviour in a fully interactive way and takes into account the changes of wind and gravity with height.

The model can be further expanded to include all aerodynamic data available for kites. The viscoelastic behaviour of the cable and simulation of the winch of the ground station can be also implemented. Results obtained using this program will help to implement the promising potential of Laddermill concept for sustainable energy production.

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