Problem of Pareto-Optimal Control for a High Altitude Energy System

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Abstract. The optimal control problem for Laddermill, a high altitude kites system for energy production is considered in this paper. The method of choice is a modification of Pareto-optimization which is extended into optimal control making a mathematically new formulation that can be employed in a wide range of optimal control problems in the field of renewable energy. Flight of the multiple kite tethered system for energy production is considered. The controlling functions are the angles of pitch, roll and yaw. Objectives are maximal energy production and stability. All approaches used in this work have necessary mathematical grounding so the solution is obtained for original multi-objective optimal control problem and it is global. Practical results of this direction of research are programs for smart remotely controlled kites for high altitude energy system.

Keywords: Laddermill, multi-objective optimal control

Introduction

The challenge of finding optimal solution from several considerations at once is one of typical tasks that every designer faces not once, and there are plenty of methods for completing such a task – genetic algorithms, adaptive control, formulation of fuzzy objective and others. However, an extension of these valuable tools into multi-objective optimal control has been announced only recently. First *Wang and Shieh* (1997) reported the extension of iterative dynamic programming for this purpose, later a fully functional approach appeared (*Podgaets et al*, 2004), (*Sarkar and Modak*, 2005). Generally speaking, such problems consist of two parts: optimal control and multi-objective optimisation. Each part can be solved with a variety of well-known methods, but there are very specific difficulties like correct linking the two approaches without losing solution's quality. This paper shows how to address a multi-objective optimal control problem in a bit more mathematically grounded way.

The problem that is solved here lies in the field of sustainable energy concept called Laddermill (*Ockels*, 1996). Laddermill consists of a dynamo that is attached to a winch, a long cable that unwinds from this winch and several kites that fly at high altitude and pull the cable (see fig. 1). When the kites ascend the cable unwinds and the winch rotates the dynamo thus generating electricity. When all rope is unwinded the dynamo spends some energy on winding the rope back. Then the cycle repeats. The dynamo, accumulators, winch and all control mechanisms and electronics form the ground station. The kites used for Laddermill are remotely controlled (smart) from the ground station. The concept promises a vast power output (*Lansdorp and Ockels*, 2005), but its actual amount depends on how smart are the smart kites.

The algorithm that is created in this research answers the following question: How to achieve the maximal power production of Laddermill and maximal speed of the rope?

The pumping Laddermill produces energy only when it's kites are going up. When the cable goes down, Laddermill is spending energy. So achieving maximal power production means maximization of this difference during the cycle.

The power is produced by the moving cable which rotates the dynamo. So apart from efficiency factor and friction in gears it is equal to the scalar product of the tension and velocity of the rope. Maximal possible speed on the way up will give less tension for the same amount of power which will lead to less heavy and costly equipment. On the contrary, the less time is spent on the way down, the better. That is why it is important to maximize the speed of descent.

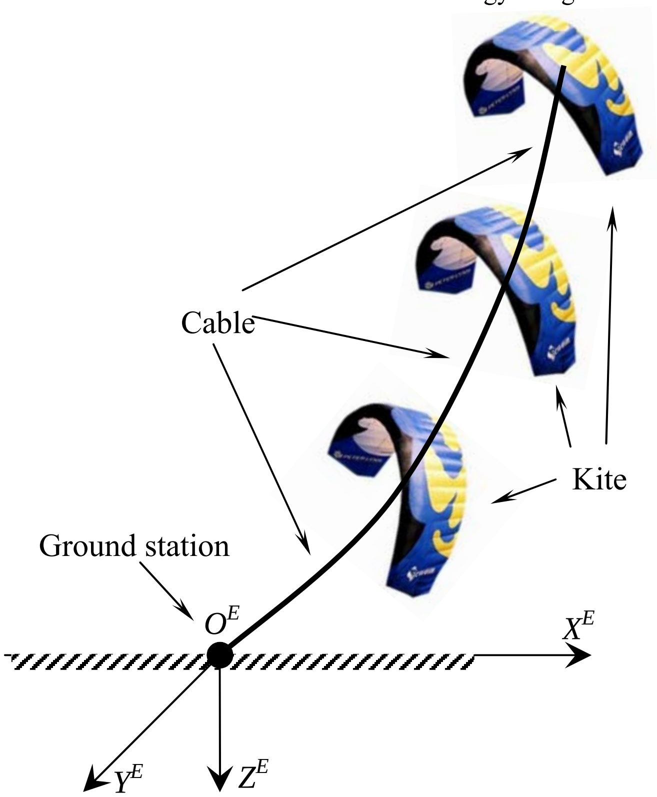

Fig. 1. The general structure of the Laddermill and Earth-fixed reference frame

Methods

Equations of motion

The equations of motion of Laddermill in the Earth-fixed reference frame (*Mulder et al*, 2006; see fig. 1) are taken from (*Podgaets, Ockels*, 2006):

$$\frac{dv_{ji}}{dt} = \frac{1}{m_j}\left(D_{ji} + L_{ji} - T_{ji} + T_{j+1i} + g_i\right), \qquad (1)$$

$$D_{ji} = -\frac{1}{2}\rho S_j c_{Dj} V_j v_{ji}, \qquad (2)$$

$$L_{ji} = \frac{1}{2}\rho S_j c_{Lj} V_j (d_{ji+1} v_{ji+2} - d_{ji+2} v_{ji+1}), \qquad (3)$$

$$T_{ji} = \frac{E_j A_j \left(R_j - R_j(t_0)\right) r_{ji}}{R_j(t_0)\sqrt{\sum_{i=1}^{3} r_{ji}^2}}, \qquad (4)$$

$$\frac{dr_{ji}}{dt} = v_{ji} + w_i, \qquad (5)$$

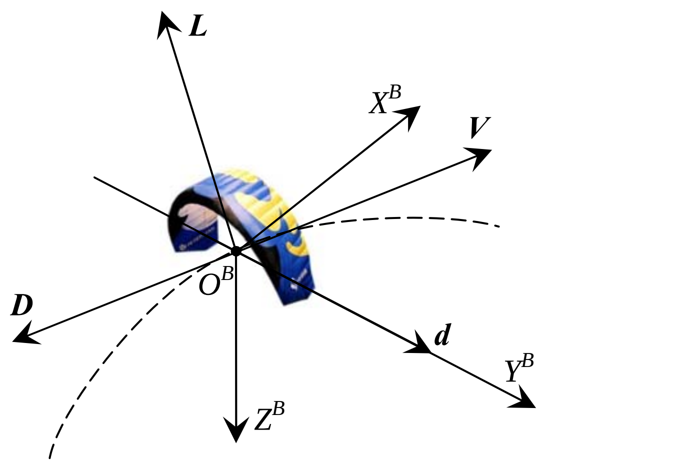

Fig. 2. Aerodynamic forces and kite's attitude in body-fixed reference frame

here j is the number of the kite (from 1 to N),
i is the number of coordinate (from 1 to 3),
$\boldsymbol{r} = (r_1, r_2, r_3)$ and $\boldsymbol{V} = (v_1, v_2, v_3)$ are the position and velocity of the kite relative to the airflow,
$\boldsymbol{R}_j = \boldsymbol{r}_j - \boldsymbol{r}_{j-1}$ is the vector pointing from the kite to the nearest element of the cable,
$\boldsymbol{w} = (w_1, w_2, w_3)$ is the wind velocity,
m, S, c_D and c_L are the kite's mass, projected area and aerodynamic coefficients,
$\boldsymbol{d} = (d_1, d_2, d_3)$ is a unit vector pointing from the left wing of the kite to the right one (see fig. 2),
$\boldsymbol{D}$, $\boldsymbol{L}$ and $\boldsymbol{T}$ are the forces of drag, lift and tension respectively (see fig. 3).

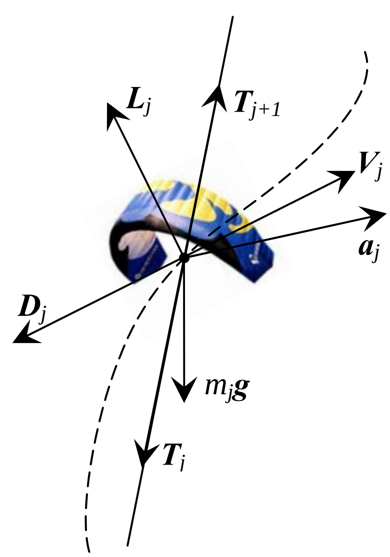

Fig. 3. Forces acting on a kite

The three attitude angles (roll ϕ, pitch θ and yaw ψ) affect the vector $\boldsymbol{d}$ pointing from the left wing of the kite to the right one:

$$\begin{cases} d_1^E = \cos\phi\cos\psi d_1^B + \cos\phi\sin\psi d_2^B - \sin\phi d_3^B, \\ d_2^E = (\sin\theta\sin\phi\cos\psi - \cos\theta\sin\psi)d_1^B + \\ + (\sin\theta\sin\phi\sin\psi + \cos\theta\cos\psi)d_2^B + \\ + \sin\theta\cos\phi d_3^B, \\ d_3^E = (\cos\theta\sin\phi\cos\psi + \sin\theta\sin\psi)d_1^B + \\ + (\cos\theta\sin\phi\sin\psi - \sin\theta\cos\psi)d_2^B + \\ + \cos\theta\cos\phi d_3^B, \end{cases} \quad (6)$$

here $\boldsymbol{d}^E$ is the vector $\boldsymbol{d}$ in the equations (1) – (5) and $\boldsymbol{d}^B = (0, 1, 0)$ is the same vector in the body-fixed reference frame (see fig. 2).

The cable is simulated as an elastic string (Sedov, 1972):

$$\frac{\partial \boldsymbol{V}}{\partial t} = \frac{\oiint\limits_{\Gamma} \boldsymbol{\sigma} \cdot d\boldsymbol{n} + \iiint\limits_{G} \boldsymbol{F} dG}{\iiint\limits_{G} \rho_{cable} dG}, \quad (7)$$

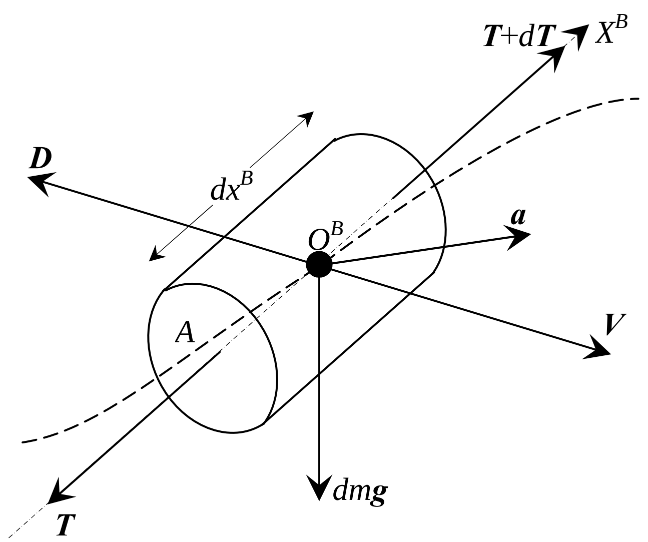

Fig. 4. Forces acting on a cable

here G and Γ are cable volume and surface respectively,

$\boldsymbol{\sigma}$ is the boundary pressure on the surface (forces of tension, aerodynamic forces),

$\boldsymbol{n}$ is the vector normal to the surface Γ and

$\boldsymbol{F}$ is the sum of the forces in the volume of the cable (gravity).

Putting the factual loads on the cable (see fig. 4) into (7) transforms it into (8)

$$dm\boldsymbol{a} = \boldsymbol{D} + \boldsymbol{T} + (\boldsymbol{T} + d\boldsymbol{T}) + dm\boldsymbol{g}, \quad (8)$$

which is then solved by finite difference method by dividing the cable between kites into N cable elements with coordinates $\boldsymbol{r}_j$ and velocity $\boldsymbol{V}_j$

$$\begin{aligned} \hat{v}_{ji} = v_{ji} + \frac{\Delta t}{m_j}\Big(D(\boldsymbol{V}_j, \alpha_j) - \\ - T(\boldsymbol{r}_{j-1}, \boldsymbol{r}_j) + T(\boldsymbol{r}_j, \boldsymbol{r}_{j+1}) + g_i(r_{j3})\Big) \end{aligned} \quad (9)$$

$$\hat{r}_{ji} = r_{ji} + \frac{\Delta t}{m_j}(v_{ji} + w_{ji}) \quad (10)$$

with the equation (2) for drag and (4) for the tension between adjacent cable elements.

Mathematical problem statement

The equations of motion (1) – (6), (9) – (10) can be rewritten in the following form:

$$\frac{d\boldsymbol{x}(t, \boldsymbol{u}, \boldsymbol{x})}{dt} = f(\boldsymbol{u}, \boldsymbol{x}), \quad (11)$$

here $\boldsymbol{x}$ is the vector of coordinates and velocities of all kite with $n = 6N$ components

$$\begin{aligned} \boldsymbol{x} = (r_{11}, r_{12}, r_{13}, r_{14}, r_{15}, r_{16}, \ldots \\ \ldots, r_{N1}, r_{N2}, r_{N3}, r_{N4}, r_{N5}, r_{N6}), \end{aligned} \quad (12)$$

$$\boldsymbol{x}(t) \in D \subset \Re^n \quad (13)$$

and $\boldsymbol{u}$ is the vector of controls

$$\boldsymbol{u}(t) = (\phi(t), \theta(t), \psi(t)), \quad (14)$$

$$\boldsymbol{u}(t) \in U \subset \Re^3. \quad (15)$$

Both control functions and parameters should be continuous along with their first derivative over time. The set of possible coordinates and velocities D dictates that all kites should be above the ground at all times and the set of possible controls U defines possible attitude angles with which kites can safely and stably fly. There are also constraints on how fast control can be executed:

$$\frac{d\boldsymbol{u}}{dt} < \boldsymbol{\varepsilon} \quad (16)$$

The constraints are taken into account directly by executing only allowed control and by excluding the wrong trajectories from further consideration. For a more detailed study of emergency cases more elaborate approach can be used.

The horizon of this optimal control problem is the end of one cycle of energy production:

$$t_0 \le t \le t_1(\boldsymbol{u}) \quad (17)$$

The objectives are energy production Φ_1 and an average velocity of the cable Φ_2:

$$\boldsymbol{\Phi} = (\Phi_1, \Phi_2) \in \Omega \subset \Re^2 \quad (18)$$

$$\Phi_1 = \int_{t_0}^{t_1} \boldsymbol{T}_0(t) \cdot \boldsymbol{V}_0(t) dt \quad (19)$$

$$\Phi_2 = \frac{1}{t_1} \int_{t_0}^{t_1} |V_0(t)| dt \quad (20)$$

$$\boldsymbol{\Phi} \to \max_{\boldsymbol{u}(t)} \quad (21)$$

The operator A transforms controls (14) into objectives (18):

$$\mathrm{A} : \boldsymbol{u} \to \boldsymbol{\Phi}, \quad (22)$$

The problem (21) with objectives (19), (20), controlling functions (14) and constraints (13), (15) and (16) is the optimal control problem with two objectives, fixed start, open end and constraints.

Method of solution

One of the ways to solve multi-objective optimal control problem is to combine two approaches – one optimal control method and one method of multi-objective optimization.

There are several approaches for numerical solution of optimal control problems – the principle of maximum (Pontryagin *et al*, 1961), methods based on the search of solution within a fixed family of functions, methods based on decomposition of controlling functions into a row over time (Fletcher, 1984), which is used here. All these methods employ the substitution of controlling functions with a set of controlling variables and further solution of resulting optimization problem. The theorem about uniqueness and existence of solution of optimal control problem is proven for such methods. The adequacy of such transition is also proven with known estimates for convergence and stability. The operator A (22) is complex and numerically calculated so its exploration has been also numerical.

Let us divide the cycle (17) into *M* steps with a constant interval Δt. Then the controlling functions can be expanded into rows over time

$$u_k(t) = \sum_{l=1}^{M} \int_{t_l}^{t_{l+1}} g_k(t) h_{kl}(t) dt, \quad (23)$$

here *k* is the number of controlling function (from 1 to 3),
l is the number of time interval (from 1 to *M*),
g_k are weight functions and
h_{kl} are shape functions that determine the solution: for each set of shape functions $\boldsymbol{H}$

$$\boldsymbol{H} = (h_{11}, h_{12}, \ldots h_{1M}, h_{21}, \ldots, h_{2M}, h_{31}, \ldots, h_{3M}), \quad (24)$$

$$\boldsymbol{H} \in C \subset \Re^{3M} \quad (25)$$

there is a unique set of objectives $\boldsymbol{\Phi}$.

Various interpolation approaches can be used to construct the functions *g* and *h*, but in the simplest case both can be constants. This case also allows the expansion to be reversed so that the original controls can be restored with any given precision, depending on the number of time intervals. This ensures the adequacy of the optimization problem

$$\boldsymbol{\Phi} \to \max_{\boldsymbol{H}} \quad (26)$$

with constraints (13), (15), (16) to the original optimal control problem (21).

Now that we expanded an optimal control problem into optimization one we need to accurately process it with some multi-objective optimization approach.

Multi-objective optimisation problems are usually solved by genetic algorithms (Sarkar and and Modak, 2005). Another popular approach is constructing a single fuzzy objective (Ali *et al*, 1997). Its belonging function usually has a meaning of "weights" of objectives and is determined by the experts. This approach is most commonly used in social sciences like economics and usually gives a good solution. Nevertheless, it has two serious drawbacks: the problem being solved is never adequate to original one, and the dialog of experts with PC is extremely obscure and thus complicated (solution totally depends on the choice of weight coefficients which does not have a direct physical meaning).

There are also heuristic approaches for finding the local optimum, which are employed in automated control systems (Zemlyakov *et al*, 1996). These methods allow full automation and are very quick but find only the nearest local peak.

Another approach to multi-objective optimisation is Pareto-optimisation (El-Ayadi, 2002 and Glumov *et al*, 2001).

Pareto-optimal set is defined as follows. A solution $\boldsymbol{u}_1$ is said to dominate over another solution $\boldsymbol{u}_2$ if and only if

$$\Phi_i(\boldsymbol{u}_1) \geq \Phi_i(\boldsymbol{u}_2) \;\; \forall\, i \in \{1,2,...,n\} \tag{27}$$

and

$$\Phi_i(\boldsymbol{u}_1) > \Phi_i(\boldsymbol{u}_2) \text{ for some } i \in \{1,2,...,n\}, \tag{28}$$

here n is the number of objectives. The solutions that are not dominated within the entire control space U (15) – and corresponding space C (25) – are denoted as Pareto-optimal and constitute the Pareto-optimal set. Following from the definition, the Pareto-set can be found by checking the points of control space C for dominating each other.

It's procedure consists of two steps:

1. Creating the table of tests. It means finding objectives Φ_i in all points of area of possible controls C. If objectives do not include exponential functions then it is possible to limit ourselves to checking some finite number of points which is determined from condition of convergence of results. The worst approach is building a grid with even intervals between points over every coordinate (Sobol, 1957). Monte-Carlo methods are usually used in planning of experiment for similar purposes of finding unknown value that depends on several parameters. Special sequences such as LP_τ (Sobol and Statnikov, 1981)

$$q_j = \sum_{j=1}^{N} \sum_{k=1}^{M} \frac{s_{kj}}{2^k} \tag{29}$$

$$M = 1 + \log_2 i \tag{30}$$

$$s_{kj} = \sum_{l=k}^{M} \frac{i r_{jl}}{2^{2l-k+3}} \tag{31}$$

produce slightly better results for fewer test points. Here $\boldsymbol{q} = (q_{i1}, q_{i2}, \ldots, q_{iN})$ is the point in the control

space C (25) – a set of controls that determines certain controlling functions (14) and certain objectives (19), (20),

i is the number of the point in the sequence,

j is the j-th coordinate of the point in C,

N is dimension of C.

If particular point is near the boundary of C then adjacent points of the boundary should be included into the table, too.

2. Determining Pareto-set by checking all test points with formulae (27) – (28).

3. Choosing constraints on objectives.

$$\boldsymbol{\Phi}^* = \left(\Phi_1^*, \Phi_2^*\right) \tag{32}$$

are the values chosen by engineer after looking at the table of tests. The points that comply with these constraints are considered the solution. They belong to the Pareto-set (27) – (28) and have the desirable values of objectives.

This algorithm is mathematically transparent and allows finding global minimum for problems with up to several dozens of objectives and practically unlimited number of controlling parameters. What is much more important, taking into account the whole area C means that the solution is a global one and is unique.

The third step can be also automated, but in the case of participation of experts the method allows organizing their dialog with computer program in an intuitively transparent way so that it is easy to use for specialists in areas other than mathematics.

For example, in the problem solved in this paper the user has to decide is how much he or she can loose in velocity of the cable at the expense of gaining in energy production. These are physical variable which meaning is clear to the user.

Results and Discussion

The results of this study are attitude angles that Laddermill kites should have in order to produce as much energy as possible with the smallest tension of the rope. They can be further used in a remote control programs on the ground station of a real Laddermill.

Conclusion

An approach for solution of multi-objective optimal control problems has been demonstrated. It includes several phases: expansion of optimal control problem into optimization problem, transporting results into multi-objective optimization procedure and it's execution. All steps are executed in a mathematically grounded way with theorems of uniqueness and existence of solution available. This ensures that convergence and given precision of the results we produce can be achieved and the results themselves are the results of originally stated problem, not some other one. The solution obtained is a global solution of a multi-objective optimal control problem for power production with Laddermill.

The same approach can be used in other renewable energy applications, as also pointed by Sarkar and Modak (2005). It allows a fully automated implementation in devices that will exert optimal control while taking into account several considerations at once.

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