FLIGHT CONTROL AND STABILITY OF A MULTIPLE KITES TETHERED SYSTEM

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One of novel concepts to use the energy of high altitude winds is by launching a series of kite on a long rope and let them pull the rope thus driving the generator. A mathematical model of tethered kites system has been developed consisting of models of kites and of the cable that links them together and to the generator on the ground energy station. The model described is then investigated for stability in various wind conditions including random wind gusts which require stochastic stability problem statement.

Keywords: Laddermill, wind energy, kites, cable dynamics, stochastic stability

INTRODUCTION

A lot of research has been done worldwide on employing aerospace technologies for sustainable development and particularly on using high altitude winds for clean energy production (e.g., [1, 14, 15]). One of novel concepts to use the energy of high altitude winds is by launching a series of kite on a long rope and let them pull the rope thus driving the generator (see figs.1, 2). This idea is called "Laddermill" and is patented by Ockels W.J. [8]. The concept has been successfully tested on a small scale with a single kite and several authors contributed to simulation of the kite systems on the rope (e.g., [6, 7, 9, 12, 13, 17]), however there is still a need for a flight control algorithm that will help to keep kites in the air and produce more energy.

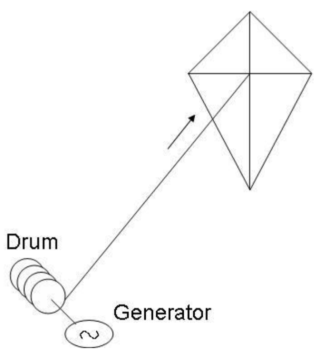

Fig.1. The concept of Laddermill

The possibility of achieving high energy production levels depends not only on overall performance of Laddermill and it's optimal control but also of optimal trajectory. That is why addressing stability is an essential part of Laddermill's mathematical description. One of the factors that affects Laddermill's performance is the wind. For example there could be a wind gust that will dramatically decrease Laddermill performance in a given moment. Even if optimal flight control is employed the controlling software will not be able to keep optimal trajectory precisely. Thus the stability of energy production has to be evaluated with stochastic distribution of parameters. The approach used for statement and solution of this problem has been first published by Gitman in 1996 [2], and in English in 1999 [10].

Fig.2. Artistic drawing of Laddermill

METHODS

Mathematical model of Laddermill

The mathematical model [12, 13] has been used (see fig. 3):

$$\frac{dv_{ji}}{dt} = \frac{1}{m_j}\left(D_{ji} + L_{ji} - T_{ji} + T_{j+1i} + g_i\right)$$

$$D_{ji} = -\frac{1}{2}\rho S_j c_{Dj} V_j v_{ji}$$

$$L_{ji} = \frac{1}{2}\rho S_j c_{Lj} V_j (d_{ji+1} v_{ji+2} - d_{ji+2} v_{ji+1})$$

$$T_{ji} = \frac{E_j A_j \left(R_j - R_j(t_0)\right) r_{ji}}{R_j(t_0)\sqrt{\sum_{i=1}^{3} r_{ji}^2}}$$

$$\frac{dr_{ji}}{dt} = v_{ji} + w_i$$

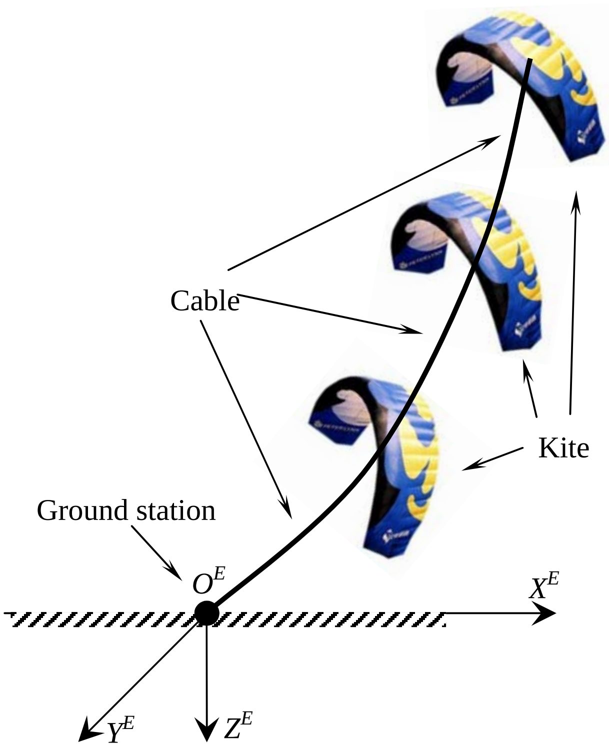

Fig. 3. The general structure of the Laddermill and Earth-fixed reference frame

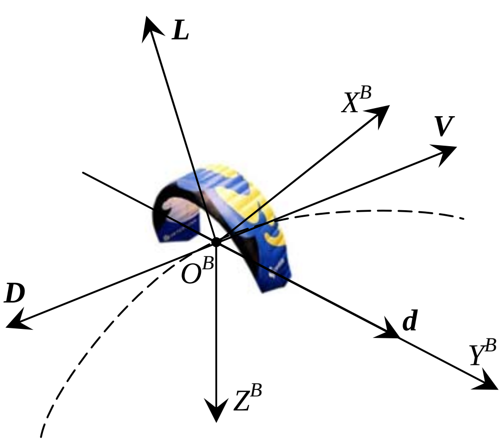

Fig. 4. Aerodynamic forces and kite's attitude in body-fixed reference frame

here j is the number of the kite (from 1 to N),

i is the number of coordinate (from 1 to 3),

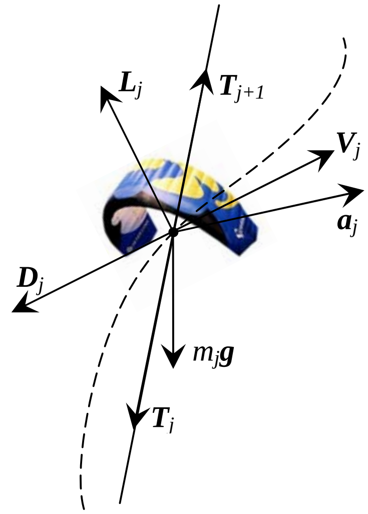

Fig. 5. Forces acting on a kite

$\mathbf{r} = (r_1, r_2, r_3)$ and $\mathbf{V} = (v_1, v_2, v_3)$ are the position and velocity of the kite relative to the airflow,

$\mathbf{R}_j = \mathbf{r}_j - \mathbf{r}_{j-1}$ is the vector pointing from the kite to the nearest element of the cable,

$\mathbf{w} = (w_1, w_2, w_3)$ is the wind velocity,

m, S, c_D and c_L are the kite's mass, projected area and aerodynamic coefficients,

$\mathbf{d} = (d_1, d_2, d_3)$ is a unit vector pointing from the left wing of the kite to the right one (see fig. 4),

D, L and T are the forces of drag, lift and tension respectively (see fig. 5).

The cable is simulated as an elastic string (fig. 6):

$$\frac{\partial \mathbf{V}}{\partial t} = \frac{\oint\!\!\oint_{\Gamma} \boldsymbol{\sigma} \cdot d\mathbf{n} + \iiint_{G} \mathbf{F} dG}{\iiint_{G} \rho_{cable} dG},$$

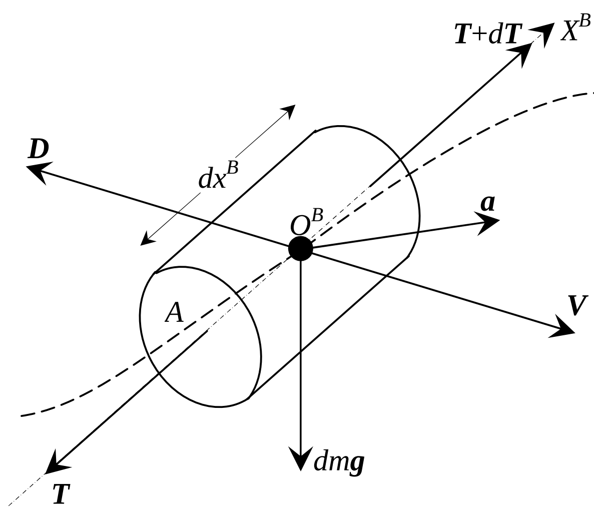

Fig. 6. Forces acting on a cable element

here G and Γ are cable volume and surface,

σ is the boundary pressure on the surface (forces of tension, aerodynamic forces),

$\mathbf{n}$ is the vector normal to the surface Γ and

$\mathbf{F}$ is the sum of the forces in the volume of the cable (gravity).

Problem of stochastic stability

Thus, the problem is following: to evaluate sensitivity of Laddermill's optimal trajectories to random deviations of wind speed. It constitutes the problem of stochastic stability of electric power to random deviations of wind speed.

Mathematical problem statement

Let us assume that Laddermill's optimal trajectory $\mathbf{x}^*(t)$ is known and programmed into the computer of ground station but somehow an unpredicted gust of wind arrives.

Let us take a large number of flight of the same Laddermill. The wind gust $\mathbf{w}$ affects the kite number j_w for the period of time T_w. The number of the kite, speed, direction and duration of wind gust are functions of a random argument that has normal distribution. According to central limit theorem wind speed $w(t)$ that affects each kite is a random function with normal distribution. Energy produced in varied wind conditions is E and energy produced in a constant wind is E^*. The index of performance of a given trajectory is a trust level

$$P^* = P\left(\left|E - E^*\right| < \varepsilon\right),$$

here ε is any number that is set beforehand. Flight trajectory is called stable if there is a level of trust $P^{**} = P^{**}\left(P^*\right)$ in closeness of input data (wind direction $\boldsymbol{\beta}_w$ and value w at each kite j) to a given value and a number $\delta(\varepsilon) > 0$ so that the trust level P^* is achieved if the following conditions are satisfied:

$$P\left(\left|\beta - \beta^*\right| < \delta_\beta\right) \geq P^{**},\ P\left(\left|w - w_0\right| < \delta_w\right) \geq P^{**}.$$

The mathematical problem statement is following: to find the trust interval of wind variation δ_w so that the energy production of a 100 kW Laddermill does not decrease more than by ε=1 kW with trust levels of $P^* = P^{**} = 0.99$. In other words, let us find

$$\delta_w \left| P\left(\left|w - w^*\right| < \delta_w\right) \geq P^{**} \Rightarrow P\left(\left|E - E^*\right| < \varepsilon\right) \geq P^* \right.$$

Algorithm of solution

1. Find representative set of energy productions. The set is called representative if

$$\frac{M_N - M_{N-1}}{\min\left(M_N, M_{N-1}\right)} < 0.1\%,$$

here M_N is an average of the set with N `elements, $N > 5$.

2. Find trust level ε of this set.

3. Achieve the given trust interval of energy production (1 kW). It is done through minimization of $\left|\varepsilon - 1kW\right|$.

4. Find resulting trust intervals of distorted parameters. Due to the fact that wind value and angle have normal distribution the trust interval for the trust level P^{**} can be found very simply using the properties of normal distribution:

$$P\left(\left|w - M(w) < k\sigma\right|\right) = 2\Phi(k) = P^{**} = 0.99,$$
$$\delta = k\sigma,$$

here $M(w)$ is an average of w,

σ is an average square variation of w and

$$\Phi(k) = \frac{1}{\sqrt{2\pi}} \int_0^k e^{-\frac{z^2}{2}} dz$$

is a Laplasian function. It is easy to find that k =2.57, so

$$\delta = 2.57\sigma.$$

The expression for trust interval of wind value is similar.

Problem of stochastic optimization

The paper [13] shows the procedure for calculation of optimal Laddermill control for maximal energy production using the approach [18]. But previous chapter shows that random gusts of wind can decrease energy production. Thus the following question can be of interest: how to fly the Laddermill so that energy production is maximal and stochastically stable if the wind has random variations.

Mathematical problem statement

$$\begin{cases} E(\mathbf{x}, \boldsymbol{\alpha}) \to \max\limits_{\boldsymbol{\alpha}} \\ P\left(\left|w - w^*\right| < \delta_w\right) \geq P^{**} \\ P\left(\left|E - E^*\right| < \varepsilon\right) \geq P^* \\ \boldsymbol{\alpha}_0 \leq \boldsymbol{\alpha} \leq \boldsymbol{\alpha}_1 \end{cases}$$

here the second and the third strings are in fact a problem of stochastic stability which in this context becomes a pair of constraints in the form of inequalities. The vectors $\mathbf{x}$ and $\boldsymbol{\alpha}$ represent the state vector of Laddermill (coordinates and velocities of all kites) and controlling angles (roll, yaw, pitch, attack angle) respectively and the last string gives constraints on operating angles.

Algorithm of solution

One of the approaches to stochastic optimization is its reduction to determinate optimization. Corresponding approach can be found in [2]. Its idea is in constructing the determinate objective that takes into account stochastic nature of certain variables. The resulting optimization problem is solved by method of deformable polygon and constraints are taken into account by method of fines.

Algorithm for calculation of optimization objective

1. Get a representative set of energy productions for given wind conditions and their variations.
2, Set the trust level of varied variable (wind speed). Achieve given trust interval of energy production with a certain precision. 1 % precision (10 W) is enough for the given application because the interval itself (1 kW) is already very small so 1 % error in it will not be noticeable. Repeat step 1 until a given trust interval is achieved.
3. Get the initial value of objective from any averaging criterion. The statistical average has been used in this work.
4. Find trust level of wind speed. Subtract the following fine from objective (for wind trust interval 5 m/s):

$$p(\Delta w) = \begin{cases} 0, & \Delta w \le 5 \\ 10(\Delta w - 5), & \Delta w > 5 \end{cases}$$

RESULTS AND DISCUSSION

The result of stochastic stability problem is a table of the following kind:

Variation of wind	Decrease in energy production

This information helps to understand how worse things can turn if the weather is unpredictable. The numbers in the right column are determinate and represent the worst possible decrease in energy production thus giving a determinate estimation of what to expect. A possibility to get exact framework for a random situation is invaluable.

The result of stochastic optimization problem is an optimal flight trajectory which is not sensitive to random wind gusts. Weather is inherently unpredictable, even on high altitudes where it becomes more stable. Thus such a “safeguard” as a robust flight trajectory is a valuable asset for energy production with Laddermill.

CONCLUSION

The methods shown in this paper are rather general and can be used in a wide variety of situations. For example, another good example of stochastic stability problem is calculation of the worst possible reduction of energy production for a conventional windmill in random wind conditions and investigation of operation of typhoon protection system under typhoon strike. The shown methods are a bit more elegant than the ones usually used because they don’t employ any procedures other than definitions of stability which makes them very transparent and strict and ensures that the answer is found for the original question.

Stochastic stability and stochastic optimization are widely used in the field of renewable energy, especially solar and wind, and this paper shows yet another easy-to-use yet strict approach for addressing these problems.

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