

# Stability of Optimal Solutions: Multi- and Single-Objective Approaches

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**Abstract**—This paper deals with assessing stability of optimal solutions. Two ways are considered: introducing stochastic stability based on Lyapunov's stability as constraints in a single-objective optimization problem and using it as a second objective. The problem from the field of wind energy is taken – optimization of electricity production with a novel wind power concept called Laddermill. Due to the multi-objectiveness of the second approach both problems are programmed with an algorithm based on a modification of Pareto-optimization. The main conclusion is that multi-objectiveness makes problem statement more transparent and also easier to implement and faster to compute which makes multi-objective formulation desirable for the class of robust optimal control problems.

several authors contributed to simulation of the kite systems (e.g., [23 – 42]) but a robust controller has not been yet published for this application.

## INTRODUCTION

The idea of this paper is to demonstrate two – single- and multi-objective – less popular ways the stability assessment can be introduced directly into optimal control, to compare and benchmark them. The following scene has been set up to make the ground even. Both approaches are implemented in the framework of the same optimization method. Due to the multi-objectiveness in the second approach the method of choice is a modification of multi-objective Pareto-optimization. And both approaches compete in solution of the same problem from the field of wind energy generation.

The structure of this paper is following: first we explain the wind energy concept that we used for benchmark, then we give mathematical statement and algorithm for solution of multi-objective problem and finally – a few remarks on the differences in a single-objective approach. The paper is finished with brief conclusions.

## MATHEMATICAL MODEL OF LADDERMILL

A lot of research has been done worldwide on employing aerospace technologies for sustainable development and particularly on using high altitude winds for clean energy production (e.g., [9, 33, 34]). The concept for sustainable energy production called Laddermill [27] (see fig. 1) is known for 10 years now [28] and refers to the system of kites on one rope that drives the generator as kites pull it. The benefits of this approach to energy production is a low weight and low cost and simplicity of the structure, installation and maintenance [26, 21]. Theoretical investigation promises capabilities of a vast power output [20]. The concept has been successfully tested on a small scale with a single kite and



Fig. 1. Artistic drawing of a Laddermill

Among recent optimization studies about kites is a design optimization paper of Peter Jackson [16], optimal control studies [15] and [3] and [41] are in different stages of preparation for publishing. Receding horizon method is used in all of them while parameters are different: yaw, lift aerodynamic coefficient and cable length in [15] and [3], roll, attack angle and cable length in [41].

There have been a lot of research in all areas of Pareto-optimization and robust optimal control. One of the first studies in optimal controllers is [17] which has been further developed in [10] and many other papers. One of the studies in robust stability is [35] which is further developed in papers

[6], [4], [19] and many others. Robust optimal control is discussed in [44] and [38]. One of the first applications of robust control for a flexible structure is [2]. Reference [18] is one of the first appearances of fuzzy logic in robust control applications. Finally one of the first genetic algorithms to utilize the concept of Pareto optimality is presented in [8].

The Laddermill is a flexible multi-body structure consisting of the kites and the cable. In the simulation of movement of the kites presented in this paper only their centres of gravity are considered. All kites on the cable have the same areas and aerodynamic coefficients and are situated at even distances in the upper part of the cable. The cable is considered elastic and the magnitude of its oscillations is considered small in respect to its length. Fully three dimensional equations of motion are used.

The equations of motion of Laddermill in the Earth-fixed reference frame (see fig. 2) are written as in [31]:

$$\frac{dv_{ji}}{dt} = \frac{1}{m_j} (D_{ji} + L_{ji} + F_{ji} - T_{ji} + T_{j+1i}) + g_i, \quad (1)$$

$$\frac{dr_{ji}}{dt} = v_{ji} + w_i, \quad (2)$$

$$D_{ji} = -\frac{1}{2} \rho S_j c_{Dj} V_j v_{ji}, \quad (3)$$

$$L_{ji} = \frac{1}{2} \rho S_j c_{Lj} V_j (d_{j+1} v_{j+2} - d_{j+2} v_{j+1}), \quad (4)$$

$$F_{ji} = \frac{1}{2} \rho S_j c_{Fj} V_j^2 d_{ji} \quad (5)$$

$$T_{ji} = \frac{E_j A_j (R_j - R_j(t_0)) r_{ji}}{R_j(t_0) \sqrt{\sum_{i=1}^3 r_{ji}^2}}, \quad (6)$$

here  $j$  is the number of the kite (from 1 to  $N$ ),  
 $i$  is the number of coordinate (from 1 to 3),  
 $\mathbf{r} = (r_1, r_2, r_3)$  and  $\mathbf{V} = (v_1, v_2, v_3)$  are the position and velocity of the kite relative to the airflow,  
 $\mathbf{R}_j = \mathbf{r}_j - \mathbf{r}_{j-1}$  is the vector pointing from the kite to the nearest element of the cable,  
 $\mathbf{w} = (w_1, w_2, w_3)$  is the wind velocity,  
 $m, S, c_D, c_F$  and  $c_L$  are the kite's mass, projected area and aerodynamic coefficients,

$\mathbf{d} = (d_1, d_2, d_3)$  is a unit vector pointing from the left wing of the kite to the right one (see fig. 3); the three attitude angles (roll  $\phi$ , pitch  $\theta$  and yaw  $\psi$ ) affect the components of vector  $\mathbf{d}$  in Earth-fixed reference frame [25],

$D, L, F$  and  $T$  are the forces of drag, lift, sideward force and tension respectively (see fig. 4).

The cable is simulated as an elastic string [37]:

$$\frac{\partial \mathbf{V}}{\partial t} = \frac{\iint_{\Gamma} \boldsymbol{\sigma} \cdot \mathbf{dn} + \iiint_G \mathbf{F} dG}{\iiint_G \rho_{cable} dG}, \quad (7)$$

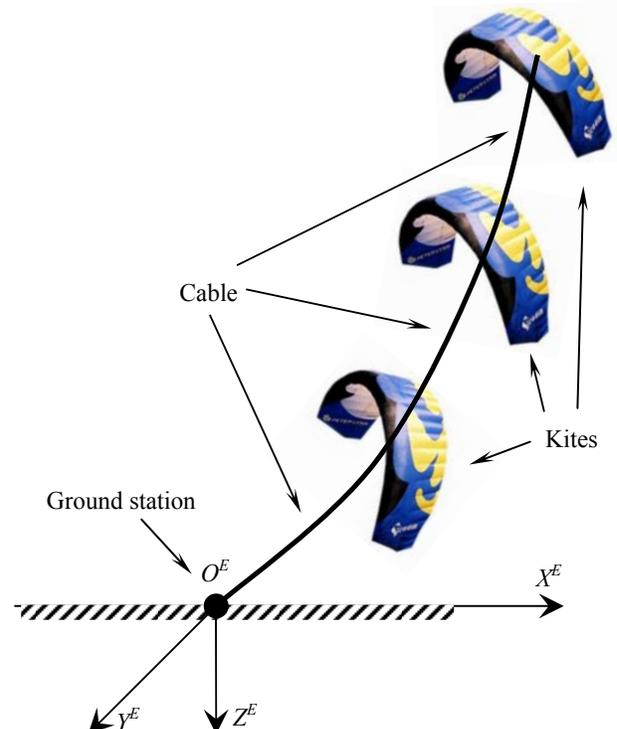


Fig. 2. The general structure of the Laddermill and Earth-fixed reference frame (index "E")

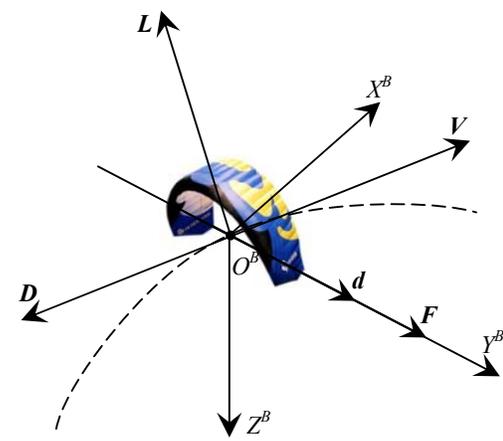


Fig. 3. Aerodynamic forces affecting the kite and kite's attitude in body-fixed reference frame (index "B")

here  $G$  and  $\Gamma$  are cable volume and surface respectively,  
 $\boldsymbol{\sigma}$  is the boundary pressure on the surface (forces of tension, aerodynamic forces),  
 $\mathbf{n}$  is the vector normal to the surface  $\Gamma$  and  
 $\mathbf{F}$  is the sum of the volume forces in the cable (gravity).

Putting the factual loads on the cable (see fig. 5) into (7) transforms it into (8)

$$d\mathbf{m}a = \mathbf{D} + \mathbf{T} + (\mathbf{T} + d\mathbf{T}) + d\mathbf{m}g, \quad (8)$$

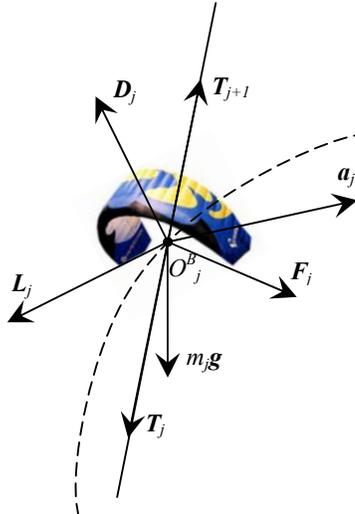


Fig. 4. Forces acting on a kite

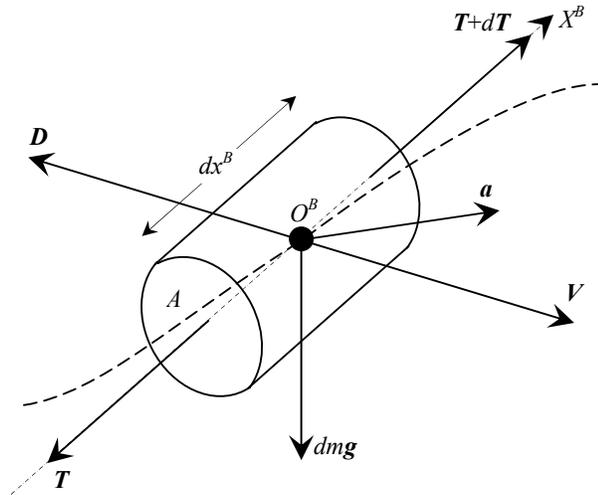


Fig. 5. Forces acting on a cable

which is then solved by finite difference method by dividing the cable between kites into  $N$  cable elements with coordinates  $r_j$  and velocity  $V_j$

$$\hat{v}_{ji} = v_{ji} + \frac{\Delta t}{m_j} (D(V_j, \alpha_j) - T(r_{j-1}, r_j) + T(r_j, r_{j+1}) + g_i(r_{j3})) \quad (9)$$

$$\hat{r}_{ji} = r_{ji} + \frac{\Delta t}{m_j} (v_{ji} + w_{ji}) \quad (10)$$

with the equation (3) for drag and (6) for the tension between adjacent cable elements.

## MULTI-OBJECTIVE APPROACH

### Mathematical problem statement

The equations of motion (1) – (6), (9) – (10) can be rewritten in the following form:

$$\frac{dx(t, u, x)}{dt} = f(u, x), \quad (11)$$

here  $x$  is the vector of coordinates and velocities of all kite with  $n = 6N$  components

$$x = (r_{11}, r_{12}, r_{13}, r_{14}, r_{15}, r_{16}, \dots, r_{N1}, r_{N2}, r_{N3}, r_{N4}, r_{N5}, r_{N6}), \quad (12)$$

$$x(t) \in D \subset \mathfrak{R}^n \quad (13)$$

and  $u$  is the vector of controls

$$u(t) = (\phi(t), \theta(t), \psi(t)), \quad (14)$$

$$u(t) \in U \subset \mathfrak{R}^3. \quad (15)$$

Both control functions and parameters should be continuous along with their first derivative over time. The set of possible coordinates and velocities  $D$  dictates that all kites should be above the ground at all times and the set of possible controls  $U$  defines possible attitude angles with which kites can safely and stably fly. There are also constraints on how fast control can be executed:

$$\frac{du}{dt} < \varepsilon \quad (16)$$

The constraints are taken into account directly by executing only allowed control and by excluding the wrong trajectories from further consideration. For a more detailed study of emergency cases more elaborate approach can be used.

The horizon of this optimal control problem is the end of one cycle of energy production which depends on control:

$$t_0 \leq t \leq t_1(u) \quad (17)$$

There will be two objective and the first objective is energy production  $\Phi_1$ :

$$\Phi = (\Phi_1, \Phi_2) \in \Omega \subset \mathfrak{R}^2, \quad (18)$$

$$\Phi_1 = \int_{t_0}^{t_1} T_0(t) \cdot V_0(t) dt, \quad (19)$$

$$\Phi_1 \rightarrow \max_{u(t)}. \quad (20)$$

*Introduction of uncertain wind*

The possibility of achieving high energy production levels depends not only on overall performance of Laddermill and it's optimal control but also on following the optimal trajectory. That is why addressing stability is an essential part of Laddermill's mathematical description. One of the factors that affects Laddermill's performance is the wind. For example there could be a wind gust that will dramatically decrease Laddermill performance in a given moment. Thus the robust control methods should be used. The way it is done here differs from classical approaches, for example [22]: we use methods that originate from evaluating stability. This approach has been first published ten years ago [13] and in English in 1999 [12].

Let us assume that Laddermill's optimal trajectory  $x^*(t)$  is known and programmed into the computer of ground station but somehow an unpredicted gust of wind arrives.

Let us take a large number of flight of the same Laddermill. The wind gust  $w$  affects the kite number  $j_w$  for the period of time  $T_w$ . The number of the kite, speed, direction and duration of wind gust are functions of a random argument that has normal distribution. According to central limit theorem wind speed  $w(t)$  that affects each kite is a random function with normal distribution. Energy produced in varied wind conditions is  $E$  and energy produced in a constant wind is  $E^*$ . The index of performance of a given trajectory is a trust level

$$P^* = P\left(|E - E^*| < \varepsilon\right), \quad (21)$$

here  $\varepsilon$  is any number that is set beforehand. Flight trajectory is called stable if there is a level of trust  $P^{**} = P^*(P^*)$  in closeness of input data (wind direction  $\beta_w$  and value  $w$  at each kite  $j$ ) to a given value and a number  $\delta(\varepsilon) > 0$  so that the trust level  $P^*$  is achieved if the following conditions are satisfied:

$$P\left(|\beta - \beta^*| < \delta_\beta\right) \geq P^{**}, \quad P\left(|w - w_0| < \delta_w\right) \geq P^{**}. \quad (22)$$

The result is a definition of stochastic stability

$$\delta_w \left| P\left(|w - w^*| < \delta_w\right) \geq P^{**} \Rightarrow P\left(|E - E^*| < \varepsilon\right) \geq P^* \right., \quad (23)$$

which can be turned around to produce the second objective for our problem. Namely, let us find such trajectory that minimizes trust interval of energy deviation  $\varepsilon$ :

$$\Phi_2 = \varepsilon \rightarrow \min_{u(t)}, \quad (24)$$

$$\begin{cases} P\left(|w - w^*| < \delta_w\right) \geq P^{**} \\ P\left(|E - E^*| < \varepsilon\right) \geq P^* \end{cases} \quad (25)$$

Here the definition (23) becomes a pair of constraints in the form of inequalities [11]. The last of them is formal and serves only for filtering out unstable trajectories.

The operator A transforms controls (14) into objectives (18):

$$A : u \rightarrow \Phi, \quad (26)$$

The problem (24) with objectives (19), (24), controlling functions (14) and constraints (13), (15) and (25) is the optimal control problem with two objectives, fixed start, open end and constraints.

*Algorithm of solution*

One of the ways to solve multi-objective optimal control problem is to combine two approaches – one optimal control method and one method of multi-objective optimization.

There are several approaches for numerical solution of optimal control problems – the principle of maximum [32], methods based on the search of solution within a fixed family of functions, methods based on decomposition of controlling functions into a row over time [7]. All these methods employ the substitution of controlling functions with a set of controlling variables and further solution of resulting optimization problem. The theorem about uniqueness and existence of solution of optimal control problem is proven for such methods. The adequacy of such transition is also proven with known estimates for convergence and stability. The operator A (22) is complex and numerically calculated so its exploration has been also numerical.

For simplicity of this part we chose finding solution in the family of functions, namely – figures “8” which can be easily implemented by controlling the yaw angular velocity (see fig. 6). The only two parameters such a control has are  $a$  for magnitude of yaw velocity switches and  $b$  for the time it is not zero. Given these two parameters we can determine entire flight trajectory:

$$H \in C \subset \mathfrak{R}^2 \quad (27)$$

Now that we expanded an optimal control problem into optimization one we need to accurately process it with some multi-objective optimization approach.

Multi-objective optimization problems are usually solved by genetic algorithms (e.g., [36]). Another popular approach is constructing a single fuzzy objective (e.g., [1]). Its belonging function usually has a meaning of “weights” of objectives and is determined by the experts. This approach is most commonly used in social sciences like economics and usually gives a good solution.

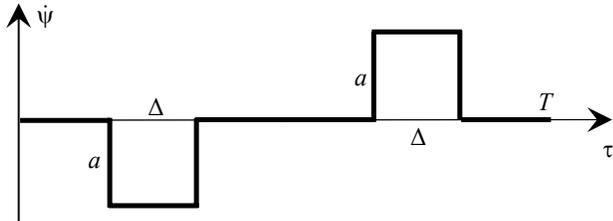


Fig. 6. Yaw control during one figure “8” of Laddermill assent. Here  $\tau$  is time during one figure “8”  $\tau \in [0, T]$  and  $T$  is it’s period.

There are also heuristic approaches for finding the local optimum, which are employed in automated control systems (e.g., [43]). These methods allow full automation and are very quick but find only the nearest local peak.

Another approach to multi-objective optimisation is Pareto-optimisation (e.g., [5, 14]).

Pareto-optimal set is defined as follows. A solution  $\mathbf{u}_1$  is said to dominate over another solution  $\mathbf{u}_2$  if and only if

$$\Phi_i(\mathbf{u}_1) \geq \Phi_i(\mathbf{u}_2) \quad \forall i \in \{1, 2, \dots, n\} \quad (28)$$

and

$$\Phi_i(\mathbf{u}_1) > \Phi_i(\mathbf{u}_2) \quad \text{for some } i \in \{1, 2, \dots, n\}, \quad (29)$$

here  $n$  is the number of objectives. The solutions that are not dominated within the entire control space  $U$  (15) – and corresponding space  $C$  (27) – are denoted as Pareto-optimal and constitute the Pareto-optimal set. Following from the definition, the Pareto-set can be found by checking the points of control space  $C$  for dominating each other. The resulting approach is slightly more mathematically correct because it uses the definition (28)-(29) directly on all points of the set  $C$  (27). During discretization a desired precision has to be preset for mapping the region, and if there are no exponential functions in the objectives the discrete problem of finding the optimal solution will be equivalent to initial one and solution will be found with given precision. Corresponding theorem can be found in [39].

Procedure of the algorithm for solution of discrete problem of multi-objective optimization consists of three steps:

1. Creating the table of tests. It means finding objectives  $\Phi_i$  in all points of area of possible controls  $C$ . If objectives do not include exponential functions then it is possible to limit ourselves to checking some finite number of points which is determined from condition of convergence of results. The worst approach is building a grid with even intervals between points over every coordinate [39]. Monte-Carlo methods are usually used in planning of experiment for similar purposes of finding unknown value that depends on several parameters. Special sequences such as  $LP_\tau$  [40]

$$q_j = \sum_{j=1}^N \sum_{k=1}^M \frac{s_{kj}}{2^k} \quad (30)$$

$$M = 1 + \log_2 i \quad (31)$$

$$s_{kj} = \sum_{l=k}^M \frac{ir_{jl}}{2^{2l-k+3}} \quad (32)$$

produce slightly better results for fewer test points.

Here  $\mathbf{q} = (q_{i1}, q_{i2}, \dots, q_{iN})$  is the point in the control space  $C$  (27) – a set of controls that determines certain controlling functions (14) and – through the mathematical model of Laddermill – certain objectives (19), (24),  $i$  is the number of the point in the sequence,  $j$  is the  $j$ -th coordinate of the point in  $C$ ,  $N$  is dimension of  $C$ .

If particular point is near the boundary of  $C$  then adjacent points of the boundary should be included into the table, too.

2. Determining Pareto-set by checking all test points with formulae (28) – (29).

3. Choosing constraints on objectives.

$$\Phi^* = (\Phi_1^*, \Phi_2^*) \quad (33)$$

are the values chosen by engineer after looking at the table of tests. The points that comply with these constraints are considered the solution. They belong to the Pareto-set (28)-(29) and have the desirable values of objectives.

This algorithm is mathematically transparent and allows finding global minimum for problems with up to several dozens of objectives and practically unlimited number of controlling parameters. What is much more important, taking into account the whole area  $C$  means that the solution is a global one and is unique.

The third step can be also automated for producing a robust controller, but in the case of participation of experts the method allows organizing their dialog with computer program in an intuitively transparent way so that it is easy to use for specialists in areas other than mathematics.

#### Algorithm for calculation of the second objective

Objective (24) requires stochastic optimization, and one of approaches to it is reduction to determinate optimization. Corresponding approach can be found in [11]. Its idea is in constructing the determinate objective that takes into account stochastic nature of certain variables. The resulting optimization problem is solved by method of deformable polygon and constraints are taken into account by penalty method. The resulting algorithm is following:

1. Get a representative set of energy productions for given wind conditions and their variations. The set is called representative if

$$\frac{M_N - M_{N-1}}{\min(M_N, M_{N-1})} < 0.1\%, \quad (34)$$

here  $M_N$  is an average of the set with  $N$  elements,  $N > 5$ .

2. Set the trust level of varied variable (wind speed).
3. Achieve given trust level and interval of energy production  $\varepsilon$ . It is done through minimization of the difference between  $\varepsilon$  and its desired value.
4. Get the initial value of objective from any averaging criterion. The statistical average has been used in this work.
5. Find trust interval of wind speed. Subtract the following fine from objective (for wind trust interval 5 m/s):

$$p(\Delta w) = \begin{cases} 0, & \Delta w \leq 5 \\ 10(\Delta w - 5), & \Delta w > 5 \end{cases} \quad (35)$$

Due to the fact that wind value and angle have normal distribution the trust interval for the trust level  $P^{**}$  can be found very simply using the properties of normal distribution:

$$P(|w - M(w)| < k\sigma) = 2\Phi(k) = P^{**} = 0.99, \quad (36)$$

$$\delta = k\sigma, \quad (37)$$

here  $M(w)$  is an average of  $w$ ,  
 $\sigma$  is an average square variation of  $w$  and

$$\Phi(k) = \frac{1}{\sqrt{2\pi}} \int_0^k e^{-\frac{z^2}{2}} dz \quad (38)$$

is a Laplasian function. It is easy to find that  $k=2.57$ , so

$$\delta = 2.57\sigma. \quad (39)$$

#### SINGLE-OBJECTIVE APPROACH

Single objective approach only maximizes energy production while regarding the quality of stability only as constraints:

$$\begin{cases} \Phi_1 = \Phi \rightarrow \max_{u(t)} \\ P(|w - w^*| < \delta_w) \geq P^{**}, \\ P(|E - E^*| < \varepsilon) \geq P^*. \end{cases} \quad (40)$$

Both multi-objective and single-objective solutions are found using the same multi-objective approach with stochastic addressing uncertainty of the wind.

#### RESULTS

The solution has been produced for the following values of parameters:

- wind is 15 m/s, directed 45° between axes  $Ox_E$  and  $Oy_E$
- air density is 1.29 kg/m<sup>3</sup> and gravity is 9.81 m/s<sup>2</sup>

- Laddermill consists of three identical kites with projected area 10 m<sup>2</sup> and mass 1 kg that are situated on the cable 5 meters from each other
- the cable has Young's modulus 1 GPa and cross-section area 1 cm<sup>2</sup> and is reeled out with a constant speed 5 m/s.

Results are given for the third kite of the Laddermill with three kites. It is done because on the last kite the movements of the third kite has the biggest magnitude for the same yaw control and it contributes the biggest part of energy produced by Laddermill. Movements of lower kites are similar although not exactly the same.

Figure 7 shows the typical yaw graph for one period, corresponding flight trajectory is presented on fig. 8.

Figures 9 and 10 show one of the competing solutions of the multi-objective problem of robust optimal control. It has a period  $T=0.28$  s, time  $\Delta=0.09$  s and yaw magnitude 93 deg.

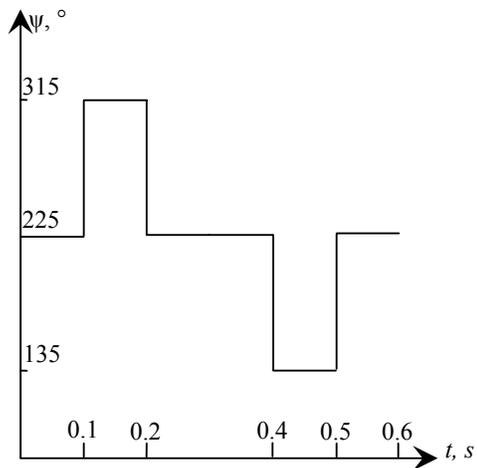


Fig. 7. One period of yaw circulation during the flight

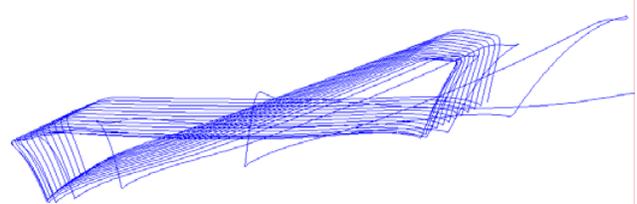


Fig. 8. Downwind view of third kite's trajectory

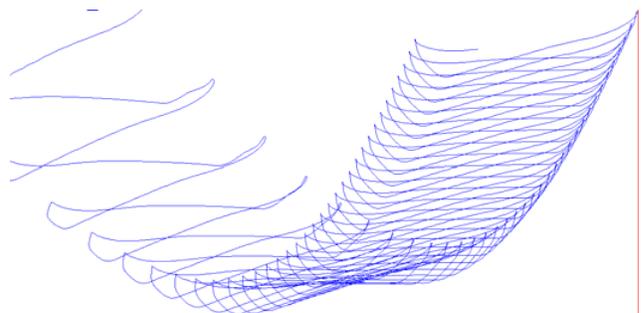


Fig. 9. Optimal trajectory of the third kite of a 3-kite Laddermill found in the multi-objective problem

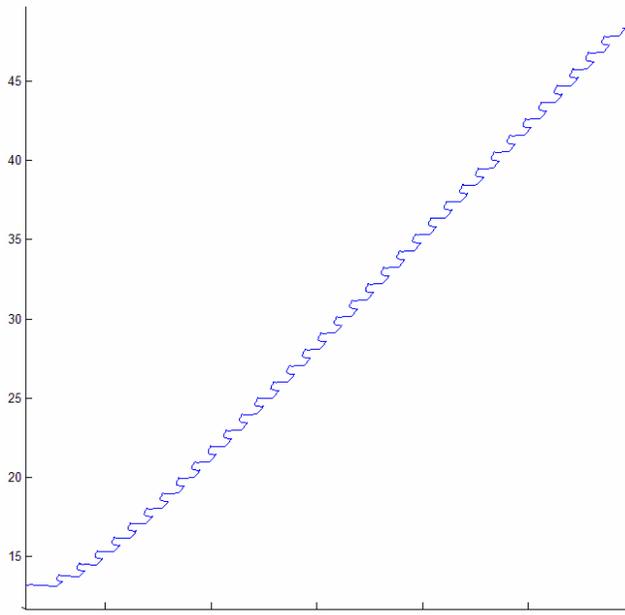


Fig. 10. Optimal trajectory of the third kite of a 3-kite Laddermill found in the multi-objective problem

Figures 11 and 12 show the solution of the single-objective problem. It has a period  $T=0.7$  s, time  $\Delta=0.3$  s and magnitude  $\alpha=90$  deg.

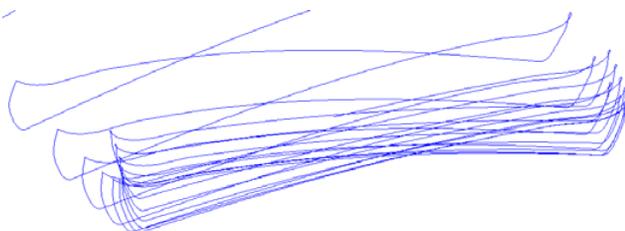


Fig. 11. Optimal trajectory of the third kite of a 3-kite Laddermill found in the single objective problem

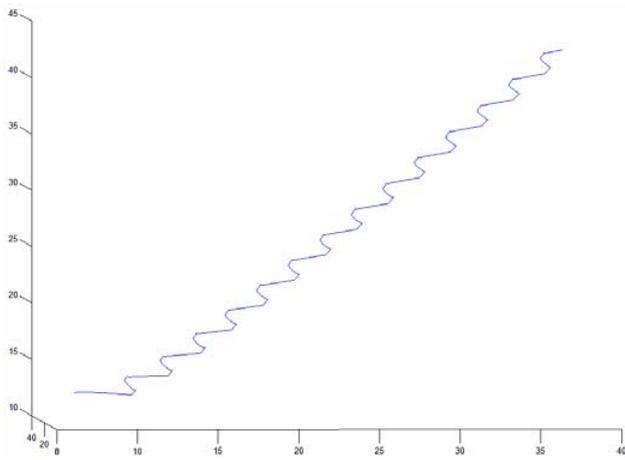


Fig. 12. Optimal trajectory of the third kite of a 3-kite Laddermill found in the single objective problem

## DISCUSSION

Although the mathematical model needs several parameters identified from experiment, some physical phenomena still can be observed despite its restrictions:

- Uncertain wind conditions proved to be not threat for a fully controllable kite: all the crashes happened solely due to a poor control, energy production mostly benefited from random gusts of wind.
- Another observation is that bigger energy production means less stability in the electric power.
- The faster are the movements of the kite the bigger is the influence of wind gusts on the change in energy production.

Solutions of both problems showed that

- the best kite trajectory has a short period
- one sharp short movement during this period is more preferable than longer more slow loop. It is unclear whether it is a universal result or a consequence of the predefine shape of control function and limitations of the model. Less constrained problem statement will answer this question.

A single-objective problem statement chose its solution because of the constraint on wind energy changes. Multi-objective problem also found this solution but disregarded it because although it is much less affected by uncertain wind, it also gives less energy.

Solution of multi-objective problem not only provided more data about the phenomenon of Laddermill behavior in uncertain wind conditions, it also takes almost twice less time because taking constraints into account is obviously more time consuming than calculation of a second objective.

## CONCLUSIONS

Two less popular methods for stating robust control has been demonstrated and compared. They are based on definitions of Pareto set and stochastic stability are using little more than that which makes them mathematically slightly more transparent and easier to ground with necessary theorems.

Both resulting problems can be – at least formally – classified as robust- multi-objective- global- optimal control. Both have been solved for the same object which has been taken from the field of wind energy (Laddermill [27]) with the same set of methods. It includes substitution of optimal control problem with optimization problem in the family of functions, transporting results into multi-objective optimization procedure and its execution. Uncertain wind conditions have been addressed by the method of stochastic optimization. All steps are executed in a mathematically grounded way with theorems of uniqueness and existence of solution available in literature. This ensures that convergence and given precision of the results we produce can be achieved and the results themselves are the results of originally stated problem, not some other one. The solution obtained is a global

solution of a multi-objective optimal control problem for power production with Laddermill.

The same approach can be used in other renewable energy applications, as also pointed by Sarkar and Modak [36]. It allows a fully automated implementation in devices that will exert optimal control while taking into account several considerations at once.

#### ACKNOWLEDGEMENTS

We are grateful to our colleagues Dr. Richard Ruiterkamp, Bas Lansdorp and Jeroen Breukels from the Aerospace for Sustainable Engineering and Technology Chair who make the Laddermill happen and Dr. Dries Visser from Design of Aircraft and Rotorcraft Chair without whom the focus of this paper would be different.

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