Comparison of two mathematical models of the kite for Laddermill sail simulation

Aleksandr Romanovich Podgaets and Wubbo Johannes Ockels

Abstract—**Laddermill sail is an innovative approach to propel the ship with the power generated by kites. The first Laddermill system is currently being designed however existing mathematical models of the system produce different optimal recommendations. Thus a decision has been made to step back and to take a closer look at the mathematical models of Laddermill sail. Each kite is considered a single rigid body as is the ship. It's been found that the differences between results might come from the fact that the two models possess features of the kite that cannot be combined in the rigid wing approach. More adequate modelling of controlling mechanisms will allow adequate modelling of Laddermill sail as a whole.**

***Index Terms*— Laddermill, Laddermill sail, kiteboat, kitesail**

I. INTRODUCTION

A lot of research has been done worldwide in using high altitude winds for clean energy production (e.g., [5, 19, 20]). The concept for sustainable energy production called Laddermill [14] (see fig. 1) is known for 11 years now [15] and refers to the system of kites on one rope that drives the generator as kites pull it. The benefits of this approach to energy production is a low weight, low cost and simplicity of the structure, installation and maintenance [13]. Theoretical investigation promises capabilities of a vast power output. The concept has been successfully tested on a small scale with a single kite and several authors contributed to simulation of the kite systems (e.g., [9, 24]) and a robust controller for this application has been recently published in [17]. This eventually led to the idea of Laddermill sail [4, 12], a ship that is propelled by Laddermill power (see fig. 2).

Recently kites has earned an increasing attention of researchers. A few methods for addressing stability of kites and parachutes are presented in [10, 21, 22]. However, being primarily design choice tools, they are not suited for robust control. Possible kite control actuators are shown in [2, 3, 8].

Fig. 1. Artistic drawing of Laddermill

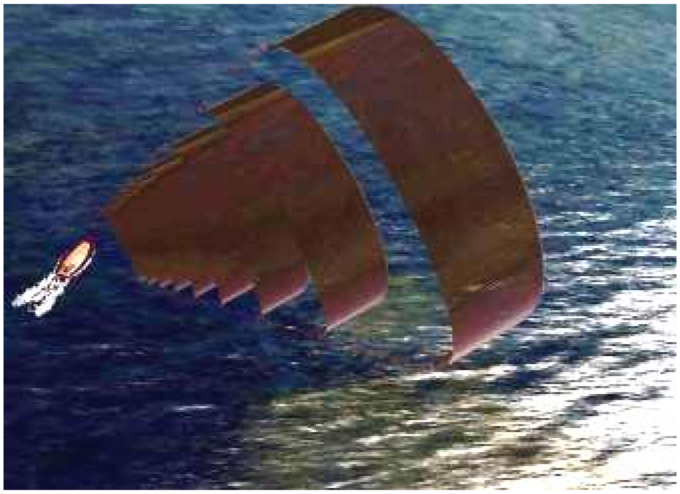


Fig. 2. Artistic drawing of a Laddermill sail [4]

Among recent optimization studies about kites is a design optimization paper [7], model-predictive control studies [6, 8], [1] and [23] are in different stages of preparation for publishing. Receding horizon and Lyapunov's parameters methods are used in all of them while control functions and optimization features are different: [1] and [23] formulate fast control for equations of motion while [6] employs Lagrange equations and full scale control, evolutionary optimization is used in [23] while [1] and [6] use multiple shooting. Control functions in [1, 23] are yaw, lift aerodynamic coefficient and cable length, and in [6] – roll, attack angle and cable length.

All listed studies treat a kite as a single rigid body with

Manuscript received July 5, 2007.

W. J. Ockels is the Chair holder of Aerospace for Sustainable Engineering and Technology, Faculty of Aerospace Engineering, Delft University of Technology, Kluyverweg 1, 2629HS, Delft, Netherlands (phone +31152789576; fax: +31152783661; e-mail: w.j.ockels@tudelft.nl, n.o.saaneh@tudelft.nl).

A.R. Podgaets is assistant professor with Aerospace for Sustainable Engineering and Technology Chair (phone: +31152785318; fax: +31152783661; e-mail: a.r.podgaets@tudelft.nl, sulion@inbox.ru).

control in the form of orientation, through roll or yaw. These two control approaches are based on different assumptions and lead to different equations of motion. So after not a few optimization attempts that did not fully agree with each other we decided to step back and examine the models themselves and understand all the features they have. After that we shall sort those features, find which are important and decide which model should be used.

II. METHODS

A. *Mathematical model of Laddermill sail*

The Laddermill sail is a flexible multi-body structure consisting of the kites, the cable and the boat. The shape of the kite does not change significantly during the flight so they are treated as rigid bodies. The cable is elastic. The equations of motion of the kites in the Earth-fixed reference frame ("E") are written as in [18]:

$$\dot{\boldsymbol{v}} = \frac{1}{m}\left(\boldsymbol{D} + \boldsymbol{L} - \boldsymbol{T}_j + \boldsymbol{T}_{j+1}\right) + \boldsymbol{g}, \tag{1}$$

$$\dot{\boldsymbol{r}}_j = \boldsymbol{v}_j, \tag{2}$$

$$\boldsymbol{D} = -\frac{1}{2}\rho S c_D V \boldsymbol{V}, \tag{3}$$

$$\boldsymbol{L} = \frac{1}{2}\rho S c_L V \boldsymbol{\Phi}^{BE}(\boldsymbol{d}) \times \boldsymbol{V}, \tag{4}$$

$$\boldsymbol{T} = \frac{EA(R - R_0)\boldsymbol{r}}{R_0 r}, \tag{5}$$

$$\boldsymbol{V} = \boldsymbol{v} - \boldsymbol{w}, \quad \boldsymbol{R}_j = \boldsymbol{r}_{j-1} - \boldsymbol{r}_j \tag{6}$$

here j is the number of the kite (from 1 to N),

i is the number of coordinate (from 1 to 3),

$\boldsymbol{r} = (r_1, r_2, r_3)$ and $\boldsymbol{V} = (v_1, v_2, v_3)$ are the position and velocity of the kite relative to the ground,

$\boldsymbol{R}_j = \boldsymbol{r}_j - \boldsymbol{r}_{j-1}$ is the vector pointing from the kite to the nearest element of the cable,

$\boldsymbol{w} = (w_1, w_2, w_3)$ is the wind velocity,

m, S, c_D, and c_L are the kite's mass, projected area and aerodynamic coefficients,

$\boldsymbol{d} = (d_1, d_2, d_3)$ is a unit vector pointing from the left wing of the kite to the right one; the three attitude angles (roll ϕ, pitch θ and yaw ψ) affect the components of vector $\boldsymbol{d}$ in Earth-fixed reference frame [11],

$\boldsymbol{\Phi}^{BE}$ is rotation matrix that converts kite's wingspan vector d from body-fixed into Earth-fixed reference frame,

$\boldsymbol{D}$, $\boldsymbol{L}$, $\boldsymbol{T}$ are the forces of drag, lift and tension respectively.

The aerodynamic coefficients of lift and drag are functions of angle of attack and represent aerodynamics of surfkite's airfoil with aspect ratio 3.

The cable between the kites doesn't play significant role so it is neglected with its parameters added to the kites. The cable between the boat and the lowest kite is treated as a single body with no lift.

The boat is considered a single rigid body and no wave or maritime processes are taken into account:

$$\dot{\boldsymbol{v}}_b = \frac{1}{m}\left(\boldsymbol{D} + \boldsymbol{T}_1 + \boldsymbol{F}(P, \gamma) + \boldsymbol{B}\right) + \boldsymbol{g} \tag{7}$$

$$\dot{\boldsymbol{r}}_b = \boldsymbol{v}_b \tag{8}$$

here $\boldsymbol{F}$ is ship's thrust, a function of power generated by kites and $\boldsymbol{B}$ is Archimedes force, a function of how deep the ship is in the water. In unlikely situation of the boat flying out of the water both are zero. The angle γ gives the course of the boat.

Equations of motion (1)-(8) are completed with initial conditions:

$$v_{j0} = v_b = 0, \quad \boldsymbol{r}_j = \boldsymbol{r}_b = 0, \quad j=1,2,\ldots,N \tag{9}$$

Ship's power curve and hull shape were taken from [16]. The boat also has a backup diesel engine: when power becomes negative it's amount is calculated separately:

$$P = T_1 v_c \tag{10}$$

$$E = -\int_{t_0}^{t_1} P\,dt, \quad \mathrm{P} < 0 \tag{11}$$

B. *Kite model with yaw control*

The kite in this model is controlled by changing its yaw angle. Surfkite tends to always be perpendicular to the cable so it's pitch and roll are governed by kite's position relative to the cable. Unfortunately neither aerodynamic coefficients nor directions of aerodynamic forces depend on yaw, so we have to introduce a link that will put yaw in control. The kite tends to turn into apparent wind so turning velocity into the current kite's forward direction (zero yaw direction) is a natural link. This rule makes a kite unstable so the turn should occur gradually. Also, the link between pitch and angle of attack has to be neglected for depowering the kite (angle of attack set to zero lift angle).

C. *Kite model with roll control*

The kite in this model is roll controlled. Roll directly influences the direction of lift. Yaw is changing according to the direction of apparent wind because the surftkite always tends to turn into flight direction, it never flies with it's wing or trailing edge forward. Depowering is realized by pitching. Unfortunately, there is no way to make kite perpendicular to the cable because all orientation angles are already occupied: roll is used for turning, pitch is used for depowering and yaw is used for always flying with leading edge forward.

D. Laddermill sail control

Laddermill control cycle is following:

1) Launch the kites
   a) Launch the first kite when time reaches t_0
   b) Reel out the rope
   c) Attach the next kite (takes some time)
   d) Repeat (a) – (c) until all kites are in the air
   e) Reel out the cable to reach operating altitude
2) Generate energy
   a) Reel out. Control follows harmonic function
   b) Reel in. Depower the kites
   c) Repeat (a) – (b) until the time is over
3) Retrieve the kites (same as launch in mirror order)
4) Stop the boat

Sometimes kites pull the boat sideward so the boat needs a feedback loop controller to follow the course. Also, kite model with yaw control always crashes if left unattended. So in this model a simple feedback loop controller is also needed for the kites. It makes the kites fly as close to zenith as possible by adding small values to yaw control that will counter it's rolling.

Controls in both models are very different so simulating absolutely the same situation is not easy. Probably the closest we can get is feeding the same parameters and control signal into both mathematical models. It should lead to very close simulations.

E. Environment

The gravity and water density (fresh water, 15 °C) are standard, air density is following International Standard Atmosphere [11] (15 °C, 101325 Pa at ground level) and wind profile over altitude is taken as a 20 years average of data from Dutch Royal Meteorological Institute (KNMI) [14]. Water currents are not taken into account.

III. RESULTS

Sample screenshot of the program is shown on fig. 3. The lowest line is trajectory of the boat, the higher lines are trajectories of the kites. In the beginning of trajectory to the right the kites make arc movements when the cable stops reeling out and it's length remains constant while the next kite is attached. There is also a small push when the cable below the kites starts reeling out. After reaching operating altitude Laddermill starts making rapid movements in the boat's lateral plane so that the tension of the cable is increased. After all cable is reeled out the kites depower and the cable is reeled in. If this phase starts when the kites are not strictly in the longitudinal plane of the ship there will be a sideward component to the tension which will pull the ship off the course. Because of a very simple boat controller (all Laddermill energy is spent directly on propulsion) boat's propeller doesn't work during reeling in and the boat returns to it's course only after the kites start producing energy again.

There have been two runs of the programs, with a smaller and larger simulation horizon. Both runs describe Laddermill sail with efficiency 50% going straight into the wind.

A. Short run

In the first (short) run the ship was standing still on an anchor and both models had the following values of parameters:

<u>Environment:</u>
Wind strength at sea level – 15 m/s
Wind angle – 0°
Simulation time step – 10^{-3} s
<u>Laddermill sail design:</u>
Boat
Laddermill sail overall efficiency – 50% (0.5)
Tonnage – 60 tons (60,000 kg)
Sc_D – 1,969 m^2
Boat stop speed – 1 knot (0.5 m/s) – end simulation condition
Boat course – 180° (straight into the wind)
Cable
Cable density – 900 kg/m^3
Cable radius – 3,5 mm
Cable stiffness – 1340625 N
Cable strength – 42000 N
Kite spacing – 15 m
Kites
Number of kites – 5
Kite mass (including cable between kites) – 5 kg
Kite area (including cable between kites – 20 m^2
Kite airfoil – surftkite with aspect ratio 3
<u>Laddermill control:</u>
Launch
Start launch time – 0 s
Launch delay, per kite – 0.2 s
Operation
Starting cable length – 50 m
Total cable length – 100 m
Reel out speed – 5 m/s
Reel in speed – -10 m/s
Period of angle control – 1 s
Magnitude of angle control – 45°
(for roll control) yaw speed – 2,25 rpm (0,235 rad/s)
(for yaw control) feedback weight – 0.01*Time step
(for yaw control) weight of velocity turn – 0.1*Time step
Parking
Parking time – 60 s
Parking delay, per kite – 0.2 s

Figs. 4-7 show Euler angles and attack angle of the highest kite. Figs. 8 and 9 show trajectory of the highest kite during two seconds in the middle of operation and its acceleration.

Obviously, all graphs exhibit significant differences the only similarity is the general shape of kite's movement. So the second, longer run has been used for looking into long-term results. Only different input parameters are listed for this run.

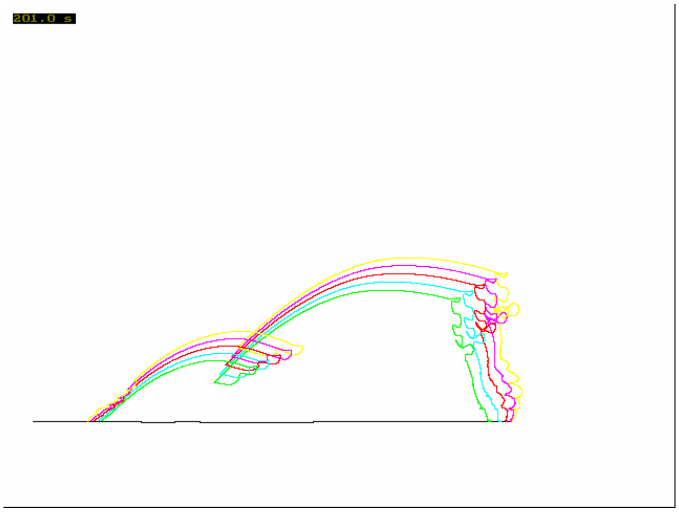


Fig. 3. Sample screenshot of the program

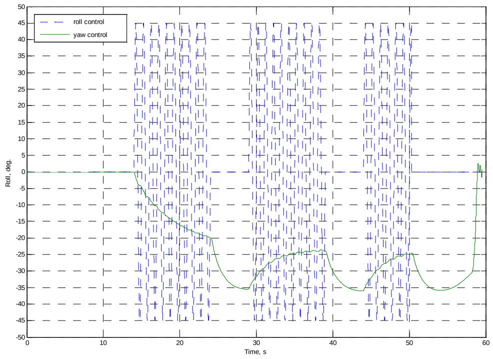


Fig. 4. Roll angle. Solid line – yaw control, dashed line – roll control

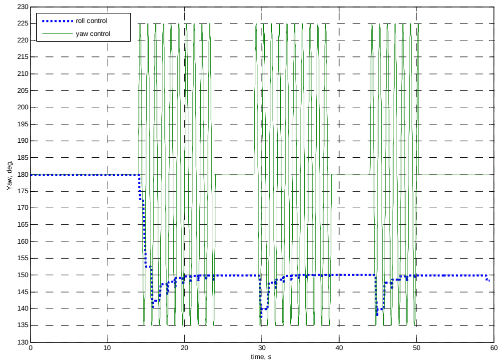


Fig. 5. Yaw angle. Solid line – yaw control, dashed line – roll control

B. Long run

Input parameters for the second run:

Laddermill control:

Launch

Launch delay, per kite – 1 s

Operation

Total cable length – 225 m

Period of angle control – 10 s

Parking

Parking time – 120 s

Parking delay, per kite – 1 s

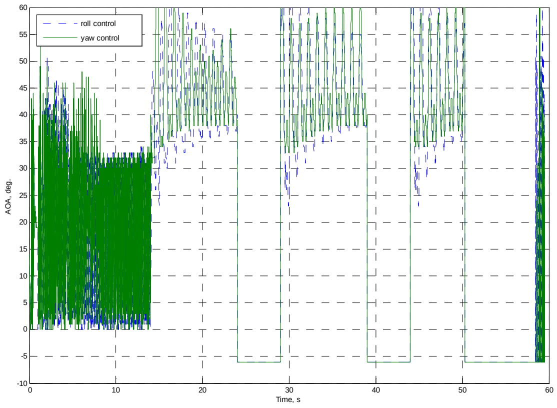


Fig.6. Angle of attack. Solid line – yaw control, dashed line – roll control

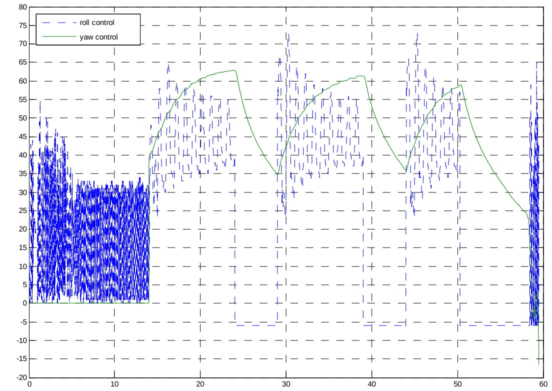
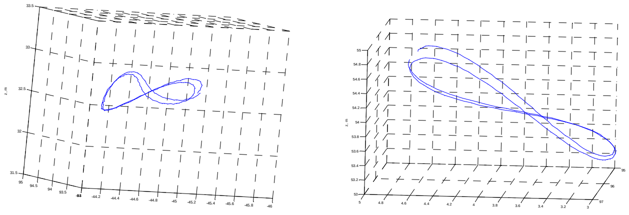
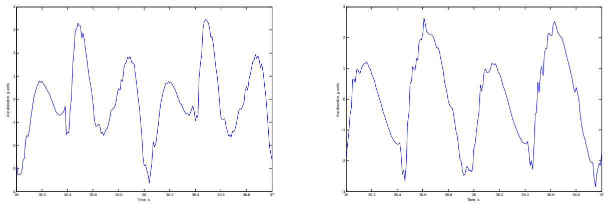


Fig. 7. Pitch angle. Solid line – yaw control, dashed line – roll control



(a) (b)

Fig. 8. Trajectory during 2 seconds. (a) yaw control, (b) roll control



(a) (b)

Fig. 9. Acceleration, g units, same two seconds. (a) yaw control, (b) roll control

Results of the second run of the two mathematical models of

Laddermill sail are listed in table 1.

Table 1. Results of the second run

	Yaw control	Roll control	ε
Diesel energy spent, MJ	2,4	1,8	25%
Distance covered, m	571,9	583,9	2%
Time of movement, s	204,73	201,09	2%
Maximal boat speed, m/s	6,53	7,24	9%
Maximal inclination angle	28,5	33,21	14%

IV. DISCUSSION

Results of comparison are quite stunning – it seems there is very little agreement between two quite similar models of Laddermill sail. This means that optimization or choice of Laddermill sail design will likely produce different results, depending on which mathematical model is used for the kite. Table 2 shows why this happens.

Table 2. Features of the kite that are represented in models

	Yaw control	Roll control
Kite is perpendicular to the cable	**Yes**, by making roll and pitch imposed by coordinates	No
Kite is unstable and requires constant micro steering	**Yes**, without feedback loop the kite always crashes	No, but it can be explained: e.g., stabilizing controller is already in place
Kite is flying only with leading edge forward	No, but it can be achieved by artificially adjusting velocity	**Yes**, yaw can track apparent wind
Kite can be depowered	No, but it can be achieved by artificially separating pitch from angle of attack	**Yes**, pitch and thus angle of attack can be set to desired values

It seems the crucial features of the kite cannot be combined in neither of the two models. The problem lies in the fact that we employ control through orientation angles. And imposing orientation on the kite is the only way of steering because of the assumption that a kite is a single rigid body.

The only way to implement all these seemingly contradicting features into one mathematical model of the surfkite is by describing controlling mechanism in sufficient detail. One of the ways to do so is separately describing left and right tip of the surfkite. This will free us from necessity to impose orientation and allow realistic modeling of kite control.

Further division of the kite into multiple bodies, “kite elements” similar to blade elements used in simulations of rotorcrafts, will improve precision of simulation.

V. CONCLUSION

Comparison of the two models of Laddermill sail has been performed in order to find more accurate one. It has been found that both models are not capable of reproducing all vital traits of kite dynamics at once. More elaborate mathematical models of controlling mechanisms will probably solve this problem.

VI. REFERENCES

[1] Canale M., Fagiano L., Ippolito M., Milanese M. Control of tethered airfoils for a new class of wind energy generators. *Conference on decision and control*, 2007 (submitted for publication)

[2] Carpenter, H.G., “Tethered Aircraft System for Gathering Energy From Wind,” US Patent 6,254,034.

[3] Chadov I.S. Stress-strain state analysis of a traction kite. Project report. Perm State Technical University, 2006

[4] Clermont F., Herman J., Horsten H., Keyifli M., Lamberts W., Peekstok A., Sap J., Venrooij J., Zuiderweg S. SAILUTION: Design of a Laddermill sail system. Final report, Design-Synthesis Exercise 2007, Group 9. TU Delft, 20 June 2007

[5] Fry C.M., Hise, H.W. *Wind driven, high altitude power apparatus*. US Patent 4084102, 1978.

[6] Houska B. Robustness and stability optimization of open-loop controlled power generating kites. MSc thesis. Ruprecht-Karls-University of Heidelberg, Faculty of Mathematics and Informatics, June 2007.

[7] Jackson P.S. Optimum loading of a tension kite. *AIAA Journal*, 2005, vol. 43, no. 11, pp. 2273-2278

[8] Ilzhoefer A., Houska B., Diehl M. Nonlinear MPC of kites under varying wind conditions for a new class of large scale wind power generators. *International Journal of Robust and Nonlinear Control* (to be published)

[9] Loyd M.L. Crosswind kite power. *Journal of Energy*, 1980, vol. 4, No. 3, 106-111

[10] Melkert J.A. STRATOW first model calculation and tests. MSc thesis. Delft University of Technology, 1992

[11] Mulder J.A., van Staveren W.H.J.J., van der Vaart J.C., de Weerdt E. *Flight dynamics lecture notes*. Delft: TU Delft, 2006.

[12] Ockels W.J. Kite power and propulsion: The Laddermill principle. *Journal of Ship Building* (submitted for publication)

[13] Ockels W.J. *Knowledge status December 1998 and baseline Laddermill test model functional requirements*. Technical note TN-O-mill 99/01, ESA: ESTEC, 1999

[14] Ockels W.J. Laddermill, a novel concept to exploit the energy in the airspace. *Aircraft Design*, 2001, 4, 81-97

[15] Ockels W.J. *Wind energy converter using kites*, November 12, 1996. European Patent EP084480, Dutch Patent NL1004508, Spanish Patent SP2175268, US Patent US6072245

[16] Ockels W.J. Private communication about a boat of Ecolutions Ltd

[17] Podgaets A.R., Ockels W.J. Flight control of the high altitude wind power system. *Proceedings of the 7th SATIS Conference* (Cape Canaveral, Florida, USA, June 3-6 2007)

[18] Podgaets A.R., Ockels W.J. Stability of optimal solutions: multi- and single-objective approaches. *CD Proceedings of IEEE Multicriteria Decision-Making Symposium*, 2007

[19] Pugh P.F. *Wind generator kite system*. US Patent 4486669, 1984.

[20] Rundle C.V. *Tethered Rotary Kite*. US Patent 5149020, 1992

[21] Sanchez G. Dynamics and control of single-line kites. *The Aeronautical Journal*, 2006, No. 9, pp. 615-621

[22] White F.M., Wolf D.F. A theory of three-dimensional parachute dynamic stability. *Journal of aircraft*, 1968, Vol. 5, No. 1, pp. 86-92

[23] Williams P., Lansdorp B., Ockels W. Modeling and control of a kite on a variable length flexible inelastic tether. *AIAA Guidance, navigation and control conference*, 2007 (submitted for publication)

[24] Williams P., Sgarioto D., Trivailo, P. Motion planning for an aerial-towed cable system. *AIAA Guidance, Navigation and Control Conference,* 2005, AIAA 2005-6267